

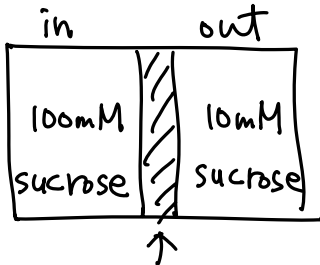
Recitation #9

6.021J 2666 Han

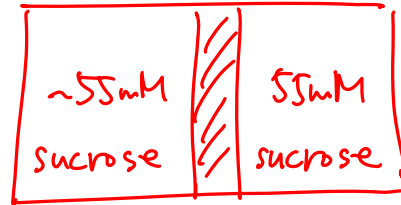
question: Describe

EQUILIBRIUM STATE.

(A)



⇒



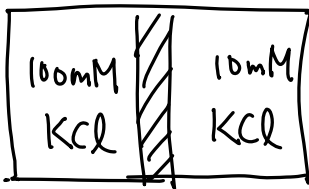
equal concentration

membrane permeable to sucrose

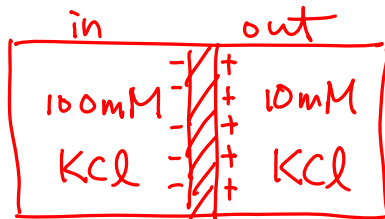
- For non-ionic substance ⇒ diffusion
- Concentration gradient ⇒ diffusive flux

$$\phi_{diff} = -D \frac{\partial C}{\partial x}$$

(B)



⇒



+ V_{K^-}

permeable to K^+ only

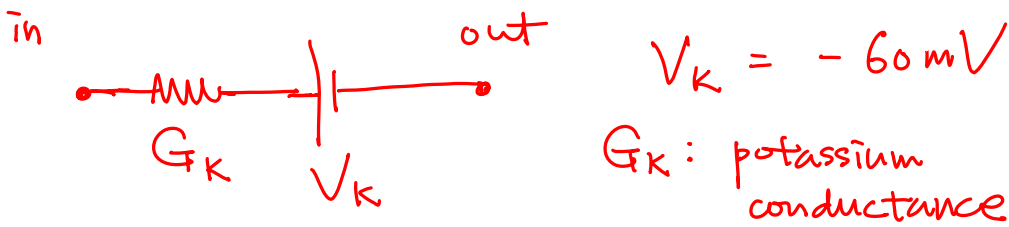
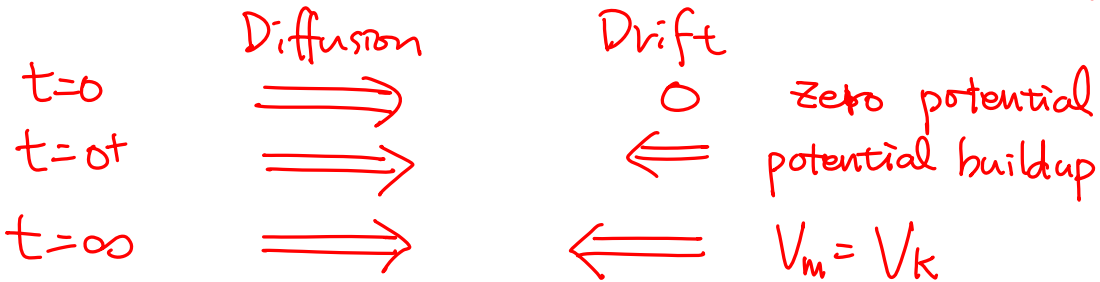
- (bulk) diffusion DO NOT happen.
- Diffusion of K^+ only ⇒ prohibited by "electroneutrality"
- Concentration gradient ⇒ Electrical potential

Nernst potential $V_K = \frac{RT}{z_n F} \ln \left(\frac{C_n^o}{C_n^i} \right)$
 (potassium potential)

$$= \frac{60mV}{z_n} \log \left(\frac{C_n^o}{C_n^i} \right)$$

Left bath (in) has lower potential.
 (Does this make sense?)

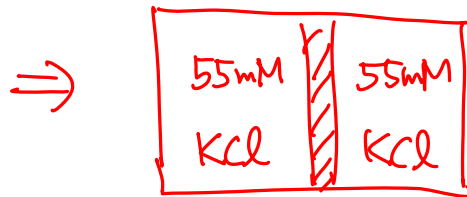
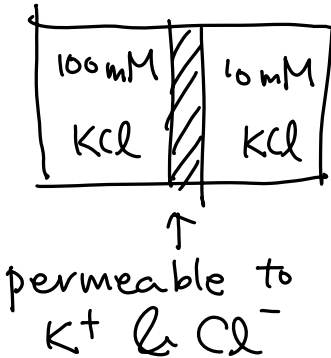
Electrodiffusive Equilibrium? YES
 $\phi_K = 0, \phi_{Cl} = 0$ (Nothing moves)



(Q) Is there any way to force equilibration?
 \Rightarrow Get rid of V_K ! (short out)

$J_m = G_K (V_m - V_K)$
 when $V_m = V_K$ (open circuit) $J_m = 0$
 $V_m = 0$ (short circuit) $J_m = -G_K V_K > 0$

(c)



"Equal concentration"
 Now K^+/Cl^- both can diffuse.

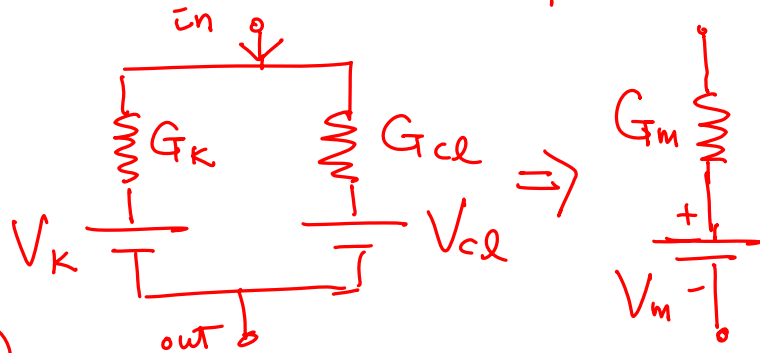
what if reservoir concentration is maintained?

\Rightarrow REST $J_m = J_K + J_{Cl} = 0$
 (quickly reached)

(quickly reached)

Why is the REST state achieved quickly?
(electroneutrality)

Circuit model



$$V_K = ? \quad (-60mV)$$

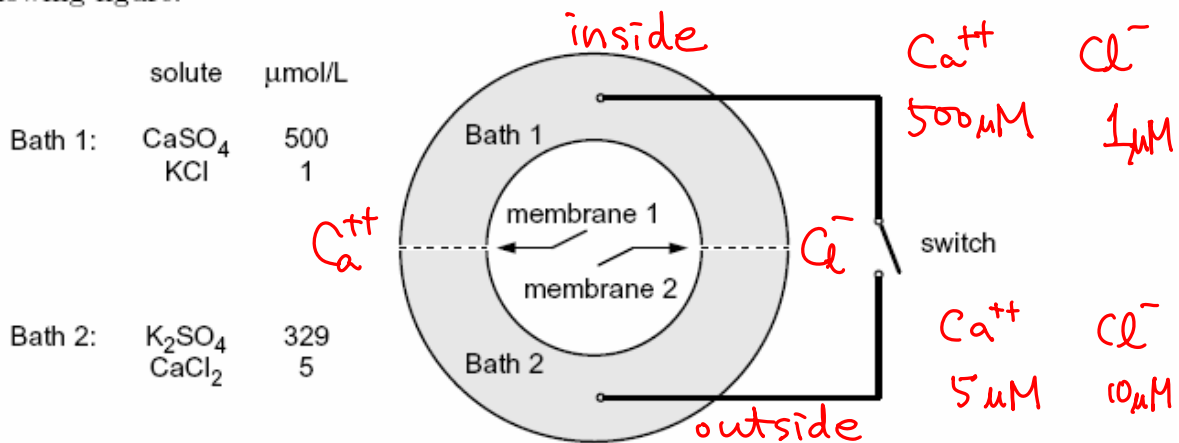
$$V_{Cl} = ? \quad (+60mV)$$

RESTING potential $V_m^0 = ?$

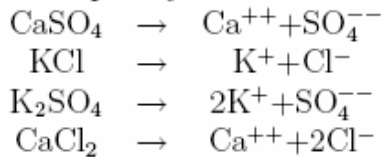
$$J_m^0 = G_K (V_m^0 - V_K) + G_{Cl} (V_m^0 - V_{Cl}) = 0$$

$$\therefore V_m^0 = \frac{G_K V_K}{G_K + G_{Cl}} + \frac{G_{Cl} V_{Cl}}{G_K + G_{Cl}}$$

A rigid toroidal chamber is divided by two rigid membranes as illustrated in the following figure.



The top and bottom parts are filled with solutions whose compositions are given in the left part of the figure. Electrodes electrically connect the top and bottom baths through a switch. Membrane 1 (left) is permeable to Ca^{++} only. Membrane 2 (right) is permeable to Cl^- only. Assume that the temperature is 300 K, and that all of the salts completely dissociate:

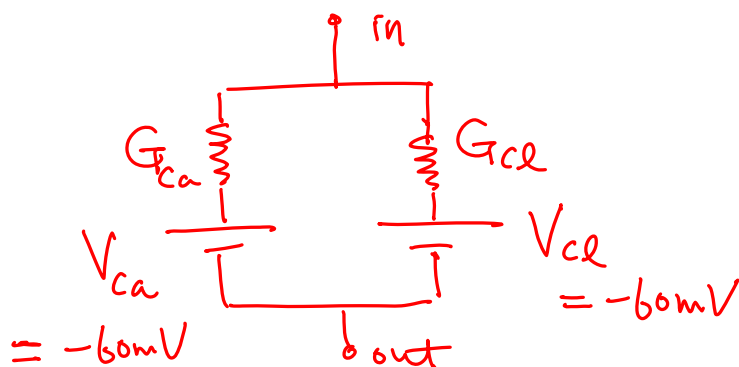


Part 1. After the switch is closed, the currents through the membranes reach steady state values within milliseconds. Is the steady state current through membrane 1 upward, downward, or zero? Explain!

$$V_{\text{Ca}} = \frac{60\text{mV}}{+2} \log_{10} \left(\frac{C_{\text{Ca}}^{\text{out}}}{C_{\text{Ca}}^{\text{in}}} \right) = 30\text{mV} \times \log_{10} \left(\frac{5}{500} \right) = -60\text{mV}$$

$$V_{\text{Cl}} = \frac{60\text{mV}}{-1} \log_{10} \left(\frac{10\mu\text{M}}{1\mu\text{M}} \right) = -60\text{mV}$$

(open circuit model)



$$\therefore V_m = -60\text{mV}$$

Membrane 1 : Ca^{++} current

$$J_{\text{Ca}} = G_{\text{Ca}} (V_m - V_{\text{Ca}})$$

when $V_m = 0 \Rightarrow J_{\text{Ca}} = -G_{\text{Ca}} V_{\text{Ca}} > 0$

$\therefore J_{\text{Ca}}$ is downward.
 Ca^{++} flows downward

Part 2. After the switch is closed, is the steady state current through membrane 2 upward, downward, or zero? Explain!

$$\text{Membrane 2} \rightarrow J_{\text{Cl}} = G_{\text{Cl}} (V_m - V_{\text{Cl}})$$
$$V_m = 0 \rightarrow J_{\text{Cl}} = -G_{\text{Cl}} V_{\text{Cl}} > 0$$

\therefore Current in membrane 2
 \rightarrow downward

$\therefore \text{Cl}^-$ flux in membrane 2 \Rightarrow UPWARD

Part 3. After the switch is opened, is the steady state flux of Ca^{++} through membrane 1 upward, downward, or zero? Explain!

Since $V_{\text{Cl}} = V_{\text{Ca}} = -60\text{mV}$, when the switch is opened this system reaches a REST,

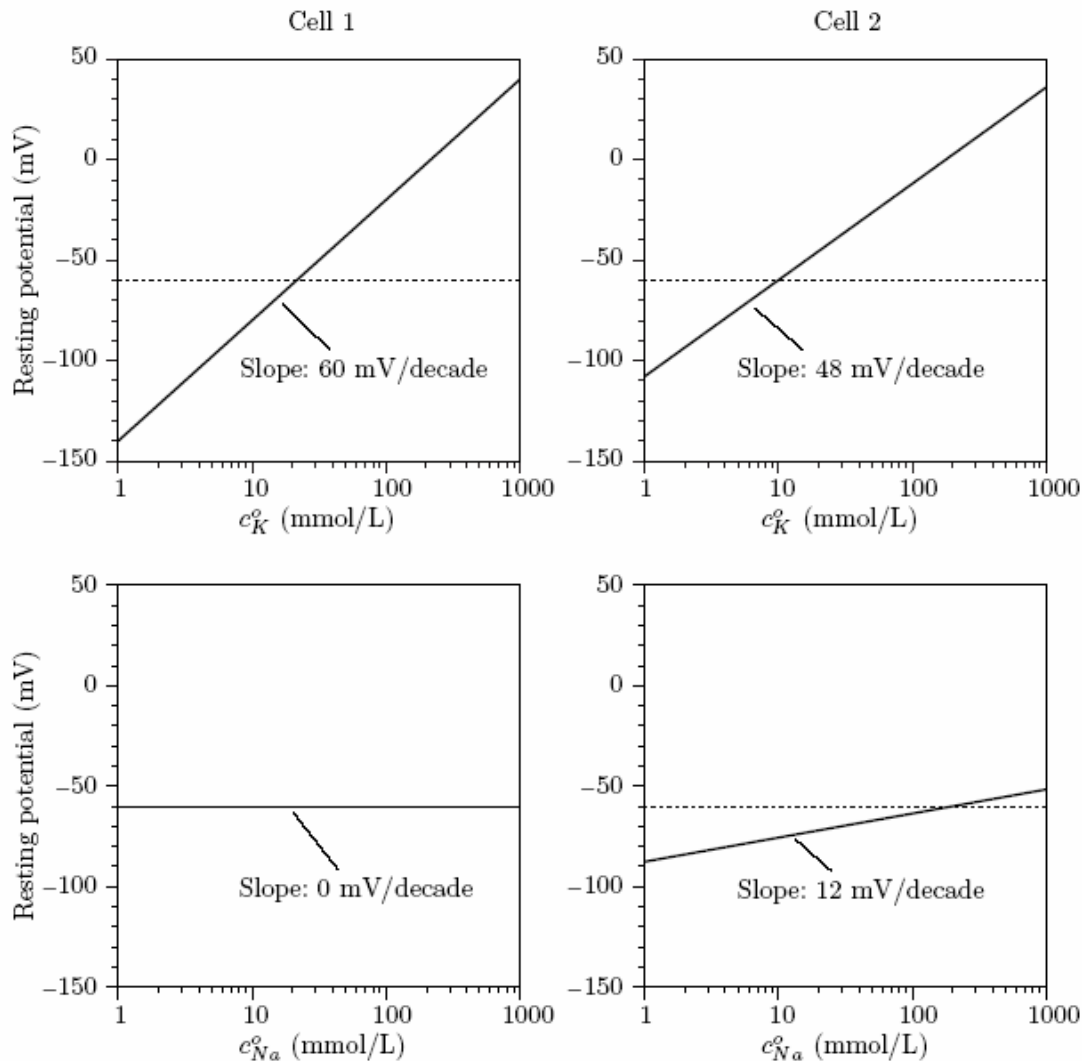
$$V_m^0 = \frac{G_{\text{Ca}} V_{\text{Ca}}}{G_{\text{Ca}} + G_{\text{Cl}}} + \frac{G_{\text{Cl}} V_{\text{Cl}}}{G_{\text{Ca}} + G_{\text{Cl}}}$$
$$= V_{\text{Ca}} = V_{\text{Cl}}$$

This is also electrodiffusive equilibrium.

since $J_{\text{Cl}} = J_{\text{Ca}} = 0$

$$\begin{array}{ccc} \uparrow & & \leftarrow \\ V_m^0 = V_{\text{Cl}} & & V_m^0 = V_{\text{Ca}} \end{array}$$

Problem 1. The resting membrane potential, V_m^o , of two uniform, isolated cells is measured as a function of the external concentration of potassium, c_K^o , with the sodium concentration held fixed at its normal value, c_{Na}^{on} , and then as a function of the external sodium concentration, c_{Na}^o , with the potassium concentration held fixed at its normal value, c_K^{on} . The results for these two cells are shown in the following figure.



You may assume that for each cell: (1) external solutions are isotonic; (2) the membranes are impermeable to ions other than potassium and sodium; (3) the internal concentrations of potassium and sodium are maintained constant by a non-electrogenic active transport mechanism; (4) the total membrane conductance is 10 nS; (5) the normal resting potential is -60 mV; (6) the internal concentration of sodium is 20 mmol/L.

(1) \rightarrow No osmotic flow

$$G_K + G_{Na} = 10 \text{ nS} = G_m$$

$$C_{Na}^i = 20 \text{ mmol/L}$$

C_{Na}^i, C_K^i : Maintained constant

- a) For each cell, determine the total conductance of the membrane to potassium and to sodium, G_K and G_{Na} , respectively. If either value is indeterminate from the information given, describe what additional information would be needed.

Remember Resting potential

$$V_m^{\circ} = \frac{1}{G_m} \sum_n G_n V_n - \sum_n \frac{J_n}{G_m} = 0$$

$$= \frac{G_K}{G_m} \frac{RT}{F} \ln \left(\frac{C_K^{\circ}}{C_K^i} \right) + \frac{G_{Na}}{G_m} \frac{RT}{F} \ln \left(\frac{C_{Na}^{\circ}}{C_{Na}^i} \right)$$

$$= \frac{G_K}{G_m} (60mV) \log_{10} \left(\frac{C_K^{\circ}}{C_K^i} \right) + \frac{G_{Na}}{G_m} (60mV) \log_{10} \left(\frac{C_{Na}^{\circ}}{C_{Na}^i} \right)$$

For Cell 1: V_m° is not dependent on C_{Na}°
 and the slope of the V_m° vs $\log_{10} C_K^{\circ} \rightarrow 60mV/dc$
 $\therefore G_K = 10nS, G_{Na} = 0$

For Cell 2:

Slope of V_m° vs $\log_{10} C_K^{\circ} \rightarrow \frac{48mV}{decade}$

$$\frac{G_K}{G_m} = \frac{48mV}{60mV} = \frac{4}{5}$$

$$G_K = 8nS$$

Similarly $G_{Na} = 2nS$.

- b) For each cell, determine the internal and external concentrations of potassium and the external concentration of sodium at the normal resting potential ($-60mV$). If the value is indeterminate from the information given, describe what additional information would be needed.

For cell 1: $C_K^{\circ} \approx 20 \text{ mmol/L}$ @ $V_m^{\circ} = -60mV$

C_{Na}° cannot be determined.

(Membrane is not permeable to Na)

From the plot, when $C_K^o \approx 200 \text{ mmol/L}$
 $V_K = 0$ ($C_K^o = C_K^i$)
 $\therefore C_K^i = 200 \text{ mmol/L}$

For Cell 2: $C_K^o \approx 10 \text{ mmol/L}$
 $C_{Na}^o \approx 200 \text{ mmol/L}$

At normal condition,

$$V_{Na} = 60 \text{ mV} \times \log_{10} \left(\frac{200}{20} \right) = 60 \text{ mV}$$

$$V_K = 60 \text{ mV} \times \log \left(\frac{10}{C_K^i} \right)$$

$$\therefore -60 \text{ mV} = \frac{4}{5} V_K + \frac{1}{5} \times 60 \text{ mV}$$

$$-120 \text{ mV} = 48 \text{ mV} \times \log \left(\frac{10 \text{ mmol/L}}{C_K^i} \right)$$

$$\therefore \frac{3}{2} = \log_{10} \left(\frac{C_K^i}{10 \text{ mmol/L}} \right)$$

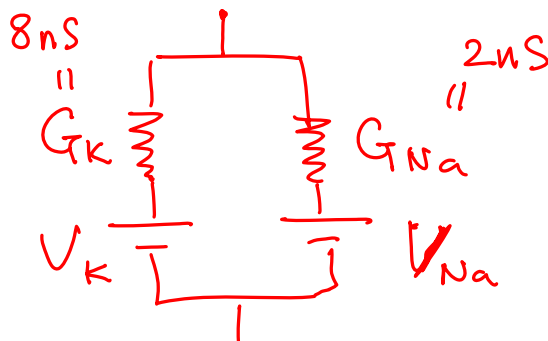
$$\therefore C_K^i \approx 316 \text{ mmol/L}$$

c) For each cell, determine the simplest equivalent electric network model that relates the dependence of the resting potential of the cell on the ion concentrations. Indicate the values of all elements in the network.

Cell 1



Cell 2.



\bar{V}_K

$\bar{V}_K \bar{V}_{Na}$