Problem 1. A long, steel pipe filled with fresh water to a height h_0 is lowered quickly into the ocean until its bottom end is a distance h_s below the surface of the ocean as shown in the Figure. The bottom end of the pipe is closed with a semipermeable membrane permeable to water only.

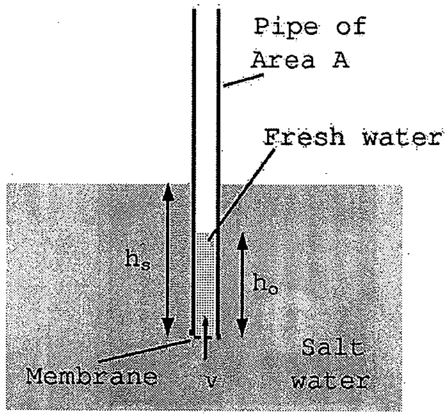

Figure 1: Schematic diagram of a pipe terminated in a semipermeable membrane and immersed in the ocean. The fresh water in the pipe has mass density ρ_0 and osmolarity 0; the salt water has mass density ρ_S and osmolarity C_Σ; the semipermeable membrane has hydraulic conductivity L_V.

a) Derive an expression for the inward flux of water, Φ_V, in terms of the variables shown in the Figure, the acceleration of gravity g, and any other necessary constants.

b) Determine the magnitude and sign of the initial derivative of $h_0(t)$, i.e., dh_0/dt evaluated at t=0, if

$h_s = 100$ m	$\rho_s = 1.03$ g/cm^3
$h_o(0) = 1$ m	$\rho_o = 1.00$ g/cm^3
$A = 10$ cm2	$L_V = 3 \times 10^{-12}$ m/(Pa·s)
$C_\Sigma = 1$ osmol/L	$g = 980$ cm/s^2
$T = 300$K	

c) Show that, provided h_s is greater than some critical depth, h_c, that the final equilibrium value of h_o is greater than h_s. Find the value of h_c.

~~[illegible]~~. Problem 1 9/21/06

a. The flux of volume is
$$\Phi_V = \mathcal{L}_V\left((p_s - p_o) - (\pi_s - \pi_o)\right).$$
The hydraulic pressure is due to the head of water and the osmotic pressure difference. The osmotic pressure of the fresh water in the pipe is zero. Therefore,
$$\begin{aligned}\Phi_V &= \mathcal{L}_V\left((p_s g h_s - p_o g h_o) - (RTC_\Sigma - 0)\right),\\ &= \mathcal{L}_V\left(g(p_s h_s - p_o h_o) - RTC_\Sigma\right).\end{aligned}$$
Note that the difference in hydraulic pressure must overcome the osmotic pressure of seawater in order for the volume flux to be positive.

b. The derivative of the height is obtained as follows
$$\frac{d(Ah_o(t))}{dt} = A\frac{dh_o(t)}{dt} = A\Phi_V.$$
Substitution for the flux yields
$$\left(\frac{dh_o(t)}{dt}\right)_{t=0} = \mathcal{L}_V\left(\rho_s g h_s - \rho_o g h_o(0) - RTC_\Sigma\right).$$
All the constants are expressed in ISI units. The difference of hydraulic pressure is
$$\begin{aligned}\rho_s g h_s - \rho_o g h_o(0) &= 9.8\,\frac{\text{m}}{\text{s}^2}\left(1.03\times 10^3\,\frac{\text{kg}}{\text{m}^3}\times 10^2\,\text{m} - 1\times 10^3\,\frac{\text{kg}}{\text{m}^3}\times 1\,\text{m}\right),\\ &= 10^6\ \text{Pa}.\end{aligned}$$
The difference in osmotic pressure is
$$\begin{aligned}RTC_\Sigma &= 8.314\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\times 300\,\text{K}\times 10^3\,\frac{\text{mol}}{\text{m}^3},\\ &= 2.49\times 10^6\ \text{Pa}.\end{aligned}$$
Therefore,
$$\left(\frac{dh_o(t)}{dt}\right)_{t=0} = 3\times 10^{-12}\,\frac{\text{m}}{\text{Pa}\cdot\text{s}}(1-2.49)\times 10^6\ \text{Pa} = -4.47\times 10^{-6}\ \text{m/s}.$$

c. At equilibrium, $\Phi_V = 0$. Therefore,
$$g\left(\rho_s h_s - \rho_o h_o(\infty)\right) - RTC_\Sigma = 0,$$
and
$$h_o(\infty) = \frac{\rho_s}{\rho_o}h_s - \frac{RTC_\Sigma}{g\rho_o}.$$
The critical height is one for which $h_o(\infty) > h_s$ so that
$$\frac{\rho_s}{\rho_o}h_s - \frac{RTC_\Sigma}{g\rho_o} > h_s,$$
and
$$\left(\frac{\rho_s - \rho_o}{\rho_o}\right)h_s > \frac{RTC_\Sigma}{g\rho_o}.$$
Therefore,
$$h_s > h_c = \frac{RTC_\Sigma}{g(\rho_s - \rho_o)}.$$

d. The critical assumption is that the ocean's composition (osmolarity) is constant with depth, i.e., that the ocean is well-mixed. If it were, then there would be no intrinsic reason that fresh water and energy could not be extracted. However, the osmolarity of the ocean increases exponentially with depth (This increase is explored more thoroughly in Problem 3.19 in volume 1 of the text). The increase results because of gravitational forces on the salts in the ocean. Over short distances relevant to physiology, these gravitational forces can be ignored. However, over long distances relevant to oceanography, they cannot.