Recitation # 1 9/7/06

First order linear (homogeneous) differential equation

$$\tau \frac{dy}{dt} + y = y_\infty$$

$$(\tau, y_\infty : \text{constants}, \quad y(t=0) = y_0)$$

Homogeneous solution

$$\tau \frac{dy}{dt} + y = 0 \Rightarrow \frac{dy}{y} = -\frac{dt}{\tau}$$

$$\Rightarrow c + \ln y = -t/\tau \Rightarrow y_h = c' e^{-t/\tau}$$

Non-homogeneous solution

$y(t) = y_h(t) + y_{n.h}$ Try $\underline{y_{n.h} = y_\infty}$ $\Rightarrow$ It works!

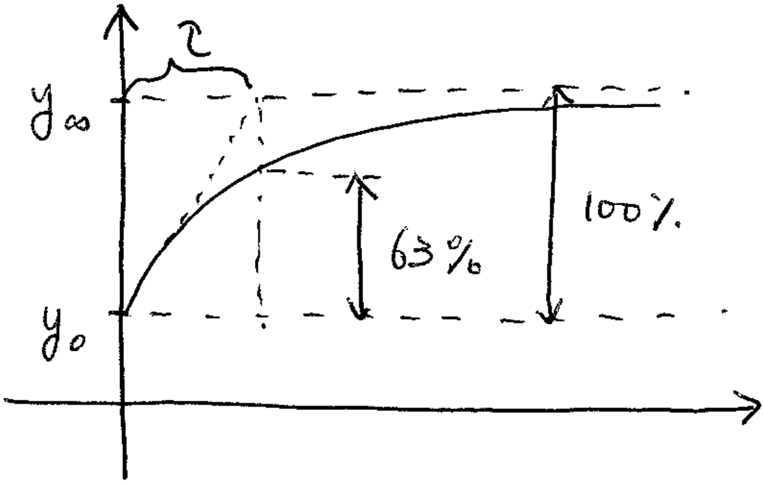
$$y(t) = c' e^{-t/\tau} + y_\infty$$

Initial condition $y(t=0) = y_0 = c' + y_\infty$

$$\therefore c' = y_0 - y_\infty$$

[Solution]

$$y(t) = (y_0 - y_\infty) e^{-t/\tau} + y_\infty$$

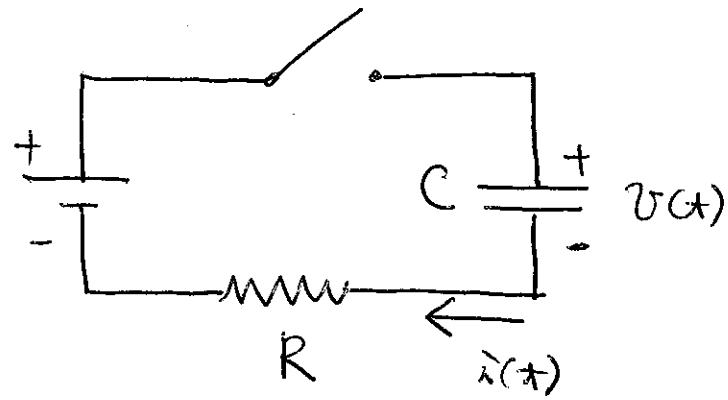


$$\left.\frac{dy}{dt}\right|_{t=0} = \frac{y_\infty - y_0}{\tau}.$$

[Example 1] RC circuit

$v(0) = 0$

at $t=0 \Rightarrow$ switch closed



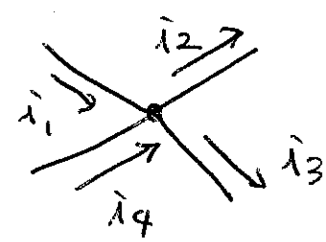
$v(t)$?

Kirchoff's voltage law

"the algebraic sum of the voltages around a loop at any instant is zero"

Kirchoff's current law.

"the algebraic sum of the currents into a node at any instant is zero"



$$\boxed{i_1 + i_4 - i_2 - i_3 = 0}$$

From the KC 1st law (voltage)

$$V_0 - v(t) - i(t)R = 0$$

From the capacitor law

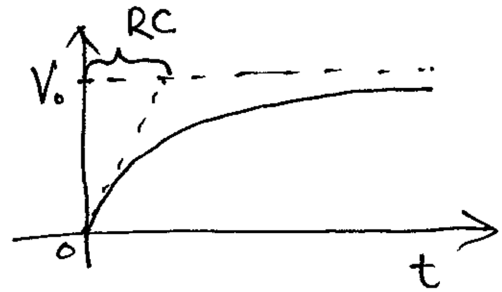
$$i(t) = \frac{d}{dt}\Big(\underbrace{Q(t)}_{\text{(charge accumulated in capacitor)}}\Big) = \frac{d}{dt}\left(C \cdot v(t)\right) \Leftarrow Q = CV$$

$$\boxed{\therefore\ RC\frac{dv(t)}{dt} + v(t) = V_0}$$

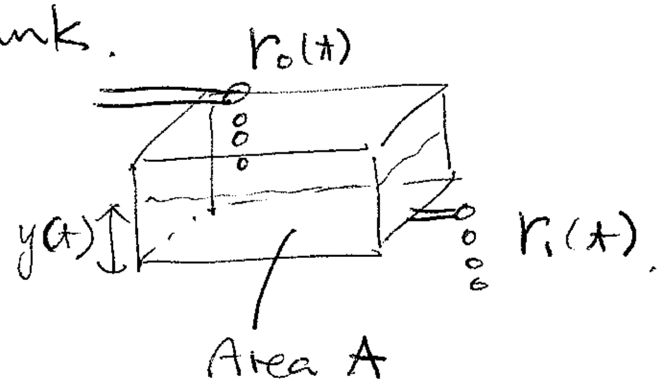
linear, 1st-order diff. eq.

$$\therefore\ \tau = RC \quad v_\infty = V_0$$

$$v(t) = V_0\left(1 - e^{-t/RC}\right).$$



[Example 2] Water tank.



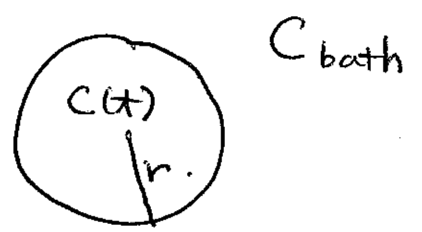
D.E. for $y(t)$?

$r_0, r_1 \Rightarrow$ volume flow rate (L/sec)

$$\frac{d}{dt}\left[\begin{matrix}\text{volume of} \\ \text{water in tank}\end{matrix}\right] = r_0 - r_1$$

$$\Rightarrow \frac{d}{dt}\left[A\,y(t)\right] = r_0(t) - r_1(t)$$

[Example 3] Spherical Cell



$$\phi = P_n\left(C_b - C(t)\right)$$

(flux) $\hookrightarrow \left[\dfrac{\text{\# of molecules transported}}{\text{Area} \cdot \text{time}}\right]$

$C(t=0) = 0$

D.E. for $C(t)$?

$$\frac{d}{dt}\left[\begin{matrix}\text{total \# of} \\ \text{molecules in the cell}\end{matrix}\right] = \left(\text{\# of molecules transported} \big/ \text{time}\right)$$

$$= \text{Flux} \times \text{Area}$$

$$\therefore \frac{d}{dt}\left[\text{Volume} \times C(t)\right] = \phi \times \text{Area}$$

$$\frac{d}{dt}\left[C(t) \cdot \frac{4\pi r^3}{3}\right] = P_n\left(C_{bath} - C(t)\right) \times 4\pi r^2$$