Quantitative Physiology: Cells and Tissues **Lecture 14: October 11, 2006 (Volume 1: 7.4)**

Nernst-Planck Equation

$$J_n(x,t) = -z_n F D_n \frac{\partial c_n(x,t)}{\partial x} - u_n z_n^2 F^2 c_n(x,t) \frac{\partial \psi(x,t)}{\partial x}$$

Continuity

$$\frac{\partial J_n(x,t)}{\partial x} = -z_n F \frac{\partial c_n(x,t)}{\partial t}$$

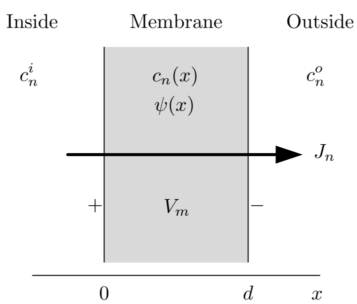
Poisson's Equation

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = -\frac{1}{\epsilon} \sum_n z_n F c_n(x,t)$$

Electrolyte solutions → Electroneutrality

$$\text{if } \; t >> \tau_r \; \text{ and } \; x >> \Lambda_D \; \text{ then } \qquad \sum_n z_n F c_n(x,t) = 0$$

Steady-State Electrodiffusion through Membranes



Steady-state

$$\rightarrow \quad \frac{\partial c_n(x,t)}{\partial t} = 0$$

$$\rightarrow \quad \frac{\partial J_n(x,t)}{\partial x} = 0$$

$$\rightarrow \quad J_n = \text{constant}$$

$$J_n = -z_n F D_n \frac{dc_n(x)}{dx} - u_n z_n^2 F^2 c_n(x) \frac{d\psi(x)}{dx} \quad = \quad -u_n z_n^2 F^2 c_n(x) \left[\frac{D_n}{u_n z_n F c_n(x)} \frac{dc_n(x)}{dx} + \frac{d\psi(x)}{dx} \right]$$

$$J_n \underbrace{\int_0^d \frac{dx}{u_n z_n^2 F^2 c_n(x)}}_{\frac{1}{G_n}} = -\int_0^d \frac{d}{dx} \left[\frac{RT}{z_n F} \ln c_n(x) + \psi(x) \right] dx$$

$$J_n \frac{1}{G_n} = -\overbrace{\frac{RT}{z_n F} \ln \frac{c_n(d)}{c_n(0)}}^{V_n} + \overbrace{\psi(0) - \psi(d)}^{V_m}$$

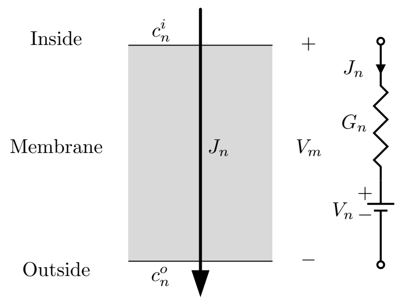
$$J_n = G_n (V_m - V_n)$$

Nernst Equilibrium Potential

$$V_n = \frac{RT}{z_n F} \ln \frac{c_n(d)}{c_n(0)} = \frac{RT}{z_n F} \ln \frac{c_n^o}{c_n^i}$$

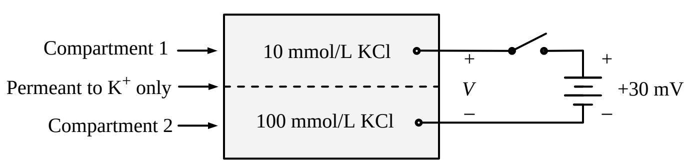
$$G_n = \frac{1}{\displaystyle\int_o^d \frac{dx}{u_n z_n^2 F^2 c_n(x)}} \geq 0$$

Model of Steady-State Electrodiffusion through Membranes



$$\text{Nernst Equilibrium Potential } V_n = \frac{RT}{z_n F} \ln \frac{c_n^o}{c_n^i}$$

$$\text{Electrical Conductivity } G_n = \frac{1}{\displaystyle\int_o^d \frac{dx}{u_n z_n^2 F^2 c_n(x)}} \geq 0$$



1. Determine the voltage V when the switch is open.
2. Determine the direction of current flow through the membrane when the switch is closed.

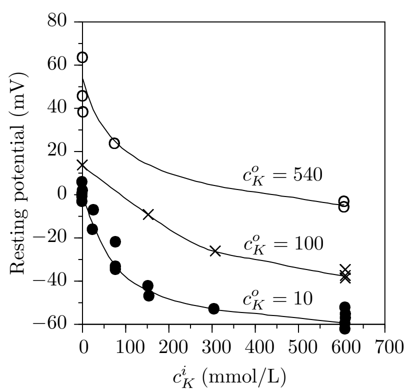


Figure 7.20