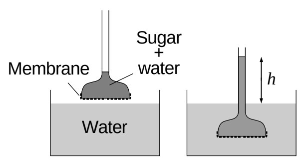
Review of Lecture 6

Osmosis = transport of solvent (water) caused by spatial differences (gradients) in solute concentration.

Osmosis can be characterized by the hydraulic pressure that would be required to stop the flow of solvent.



van't Hoff's law: osmotic pressure can be calculated using the ideal gas law -- just substitute the number of solute particles for the number of gas molecules:

$$\pi(x,t) = RT \sum_n C_n(x,t) = RT\, C_\Sigma(x,t)$$

Water transport by hydraulic pressure

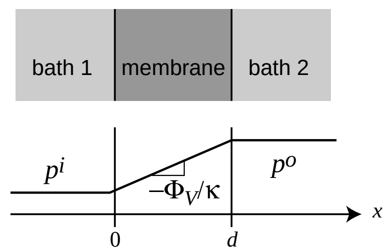
Darcy's law:

$$\Phi_V(x,t) = -\kappa \frac{\partial p(x,t)}{\partial x}$$

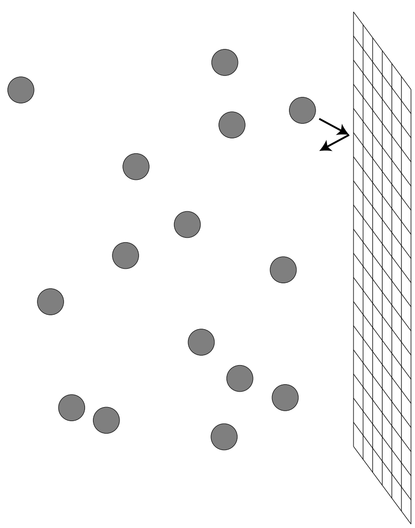
Continuity

$$-\frac{\partial}{\partial x}\left[\rho_m(x,t)\Phi_V(x,t)\right] = \frac{\partial \rho_m(x,t)}{\partial t}$$

Water flow through semipermeable membrane



$$\Phi_V = \mathcal{L}_V\,(p^i - p^o)$$



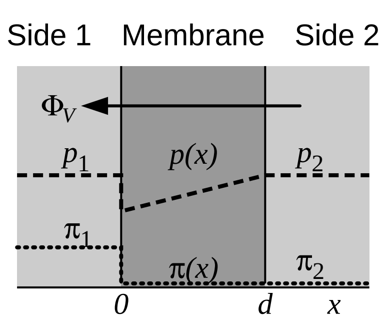
solute collides with mesh

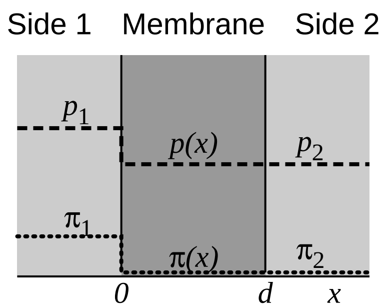
mesh exerts force on solute
→ changes solute momentum

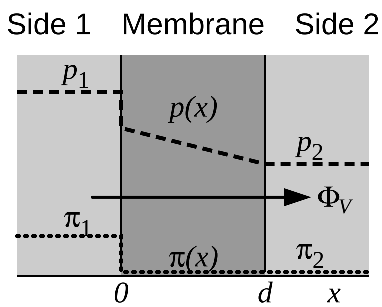
solute collides with solvent
→ transfers momentum to solvent

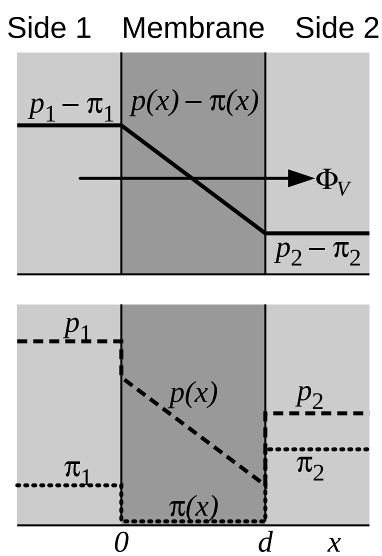
change in solvent momentum
is equivalent to a hydraulic pressure

change in hydraulic pressure
= change in osmotic pressure

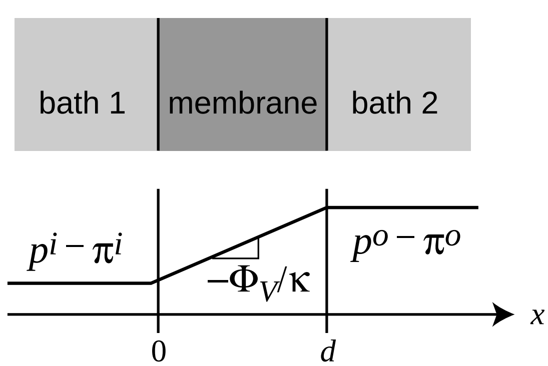






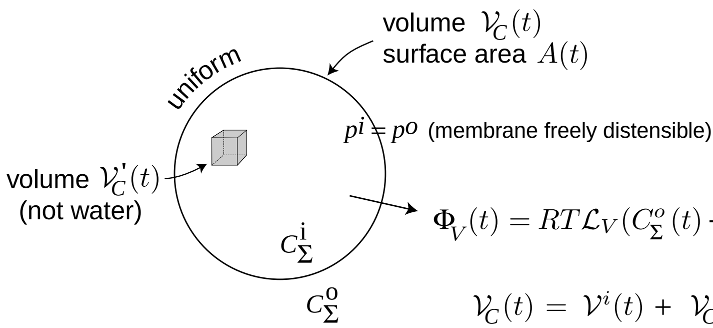


Water flow through semipermeable membrane



$$\Phi_V = \mathcal{L}_V\left[(p^i - \pi^i) - (p^o - \pi^o)\right]$$

Primary osmotic responses of cells
(flow of water due to change in concentration of impermeant solute)



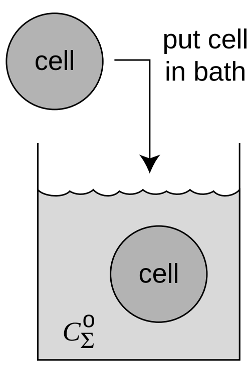
$$\Phi_V(t) = RT\mathcal{L}_V\left(C_\Sigma^o(t) - C_\Sigma^i(t)\right)$$

$$\mathcal{V}_C(t) = \mathcal{V}^i(t) + \mathcal{V}_C'(t)$$

Conservation of water:

$$\frac{d\mathcal{V}^i(t)}{dt} = -A(t)\,\Phi_V(t) = -A(t)RT\mathcal{L}_V\left(C_\Sigma^o(t) - C_\Sigma^i(t)\right)$$

Equilibrium Solution

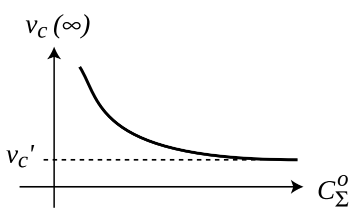


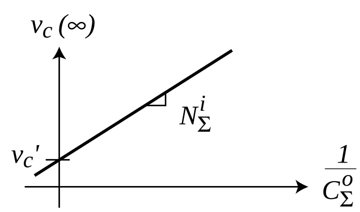
EQ: $\dfrac{\partial v^i}{\partial t} = 0 \quad \rightarrow \quad C_\Sigma^o(\infty) = C_\Sigma^i(\infty)$

If membrane impermeant to all solutes
then total solute inside cell cannot change

$$N_\Sigma^i = C_\Sigma^i(t)\,v^i(t) = \text{constant}$$
$$= C_\Sigma^i(\infty)\,v^i(\infty)$$
$$= C_\Sigma^o\,v^i(\infty)$$
$$v^i(\infty) = \frac{N_\Sigma^i}{C_\Sigma^o} = v_c(\infty) - v_c'$$
$$v_c(\infty) = v_c' + \frac{N_\Sigma^i}{C_\Sigma^o}$$

$\rightarrow$ perfect osmometer





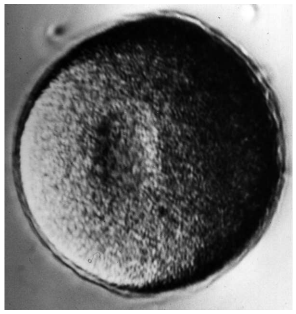


Figure 4.20

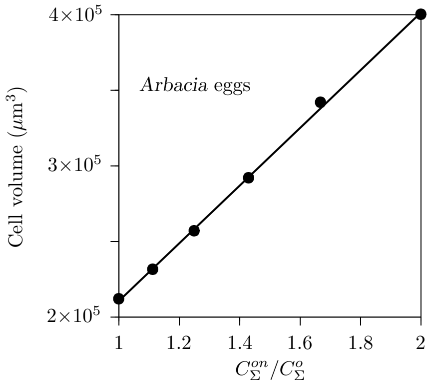


Figure 4.21

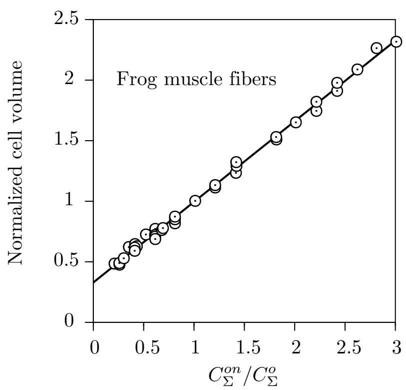


Figure 4.22

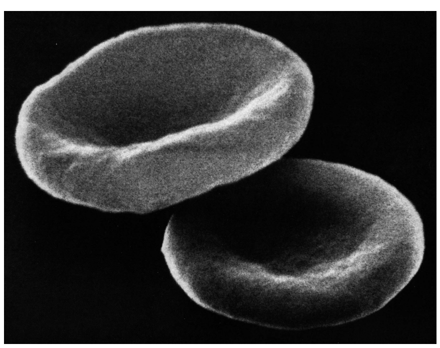


Figure 4.23

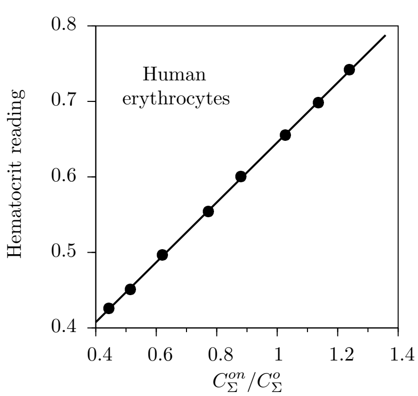


Figure 4.24