Intracellular
Extracellular
Membrane

Dissolution and diffusion through lipid bilayer
Transport through water channels
Transport through gated ion channels
Carrier-mediated transport
Pumps

Figure 2.19

2.5 μm

Figure 1.1

Water transport in digestive system

Daily traffic
- 800 g food + 1.2 L water ingested daily
- 1.5 L saliva
- 2 L gastric secretions
- 0.5 L bile
- 1.5 L pancreatic secretions
- 1.5 L intestinal secretions
- 7 L digestive fluids
- 15 pounds of water (10% of body weight) secreted and reabsorbed daily

Salivary glands
Pharynx
Esophagus
Trachea
Liver
Stomach
Pancreas
Small intestine
Large intestine
Rectum
Anus

Osmosis Observations

Henri Dutrochet (early 1800s)
- first described phenomenon and called it osmosis
- developed first osmometer: animal bladder filled with test solution, plunge into water, swells, turgid
- pressure greater for solutions with more solute
Wilhelm Pfeffer (mid 1800s)
• osmosis can be stopped with hydraulic pressure
• thistle tube + animal bladder (or artificial membrane by late 1800s)
  – water flows in direction to equalize sugar concentration
  – hydraulic pressure develops
  – flow stops when osmotic pressure = hydraulic pressure
• pressure proportional to concentration of solute
• pressure increases slightly with temperature

Hugo de Vries (late 1800s)
• studied osmosis in cells
• animal cell can shrink or swell depending on concentration
• isotonic (same “tension” as in cell’s normal environment)
• plasmolysis – plant cell membrane separates from cell wall
• except for salts, plasmolysis occurs at same MOLAR concentration
  (does not depend on chemical properties of solute)
  → colligative property (freezing point depression, boiling point elevation)
• salts are different: ratios of small integers

Henricus van't Hoff (1886)
• formulated mathematical law
• count number of particles in volume V
• measure temperature T
• osmotic pressure = pressure produced by gas with same
  number of particles, same volume, and same pressure

\[
\pi(x,t) = RT \sum_{n} C_n(x,t)
\]

van't Hoff's Law

• salts are different
Svante Arrhenius (1884)
- PhD (age 25): dissolution of salts into ions
- NaCl $\rightarrow$ Na$^+$ + Cl$^-$ (\(\therefore\) conducts electricity)
- count ions as separate particles
  $\rightarrow$ van’t Hoff’s law works for salts as well

\[ \pi(x,t) = RT \sum_n C_n(x,t) = RT \Sigma C\Sigma(x,t) \]
\[
\text{osmotic pressure} \quad \text{osmolarity}
\]
[ Pa = N/m$^2$ ]

**Summary**
- Osmosis can be characterized by a pressure that is given by van’t Hoff’s Law:
  \[ \pi = RT \sum_n C_n = RT C\Sigma \]
- Osmotic pressures cause water to flow
  ... similar to the flow caused by hydraulic pressure
  ... but in the opposite direction!
- Water flow stops when osmotic pressure difference is equal to the hydraulic pressure difference:
  \[ p_1 - p_2 = \pi_1 - \pi_2 \]

$\begin{array}{|c|c|}
\hline
\pi_1 & \pi_2 \\
\hline
\end{array}$

**Controversy**
- no question that van’t Hoff’s law is true
- but why?

- why should water go TOWARD the solute?
- large osmotic pressure ATTRACTS water!
solute collides with mesh
mesh exerts force on solute
→ changes solute momentum
solute collides with solvent
→ transfers momentum to solvent
change in solvent momentum is equivalent to a hydraulic pressure
change in hydraulic pressure = change in osmotic pressure

Macroscopic laws of solvent transport: hydraulic case

Darcy’s Law: flow through porous medium

\[
\Phi_V(x,t) = -\kappa \frac{\partial p(x,t)}{\partial x}
\]

- analogous to Fick’s law for diffusion, Ohm’s law for electrical current, Fourier’s law for heat flow
- units for solvent flux different from units for solute flux

\[
\Phi_n \left[ \frac{\text{mol}}{\text{m}^2\text{s}} \right] \quad \Phi_V \left[ \frac{\text{m}^3}{\text{m}^2\text{s}} = \frac{\text{m}}{\text{s}} \right] \quad p \left[ \text{Pa} = \frac{\text{N}}{\text{m}^2} \right]
\]

Continuity Equation for Solvent Flow

Net mass of solvent entering through edges during \((t, t+\Delta t)\)

\[
\rho_m(x,t+\Delta t) \frac{\Delta \Phi_V(x,t)}{\Delta x} = \rho_m(x+\Delta x,t) \frac{\Delta \Phi_V(x+\Delta x,t)}{\Delta x}
\]

equal if solvent is neither created nor destroyed

\[
\frac{\rho_m(x,t+\frac{1}{2}) \Phi_V(x,t+\frac{1}{2}) - \rho_m(x+\Delta x,t+\frac{1}{2}) \Phi_V(x+\Delta x,t+\frac{1}{2})}{\Delta x} = \frac{\rho_m(x,\frac{1}{2},t+\Delta t) - \rho_m(x+\Delta x,\frac{1}{2},t)}{\Delta t}
\]

Take limit as \(\Delta x \to 0\) and \(\Delta t \to 0\)

\[
\frac{\partial}{\partial x} \left[ \rho_m(x,t) \Phi_V(x,t) \right] = \frac{\partial \rho_m(x,t)}{\partial t}
\]
Macroscopic laws of water transport (hydraulic case):

- Darcy's law:
  \[ \Phi_{V}(x,t) = -\kappa \frac{\partial p(x,t)}{\partial x} \]

- Continuity
  \[-\frac{\partial}{\partial x} \left[ \rho_m(x,t) \Phi_{V}(x,t) \right] = \frac{\partial \rho_m(x,t)}{\partial t} \]

Assume water is incompressible
\[ \rho_m(x,t) = \rho_0 = \text{constant in space and time} \]
\[ -\rho_0 \frac{\partial}{\partial x} [\Phi_{V}(x,t)] = 0 \]
\[ \rightarrow \Phi_{V}(x,t) = \text{constant in space} \]
\[ \rightarrow p(x,t) = \text{linear function of space} \]

very much like SS case for diffusion
but always true here since water is incompressible

Water flow through thin membrane
(hydraulic)

\[ \Phi_{V}(t) = \mathcal{L}_{V} (p^i(t) - p^o(t)) ; \quad \mathcal{L}_{V} = \kappa / d \]

hydraulic conductivity

analogous to Fick's law for membranes
\[ \phi_n(t) = P_n (c^i_n(t) - c^o_n(t)) ; \quad P_n = \frac{D_n k_n}{d} \]