# Quantitative Physiology: Cells and Tissues

**Lecture 6: September 18, 2006**
**(Volume 1: 4.1-4.3.2.3; 4.4-4.5.1.2)**

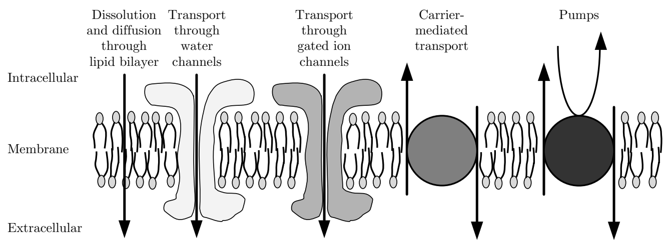



Figure 2.19

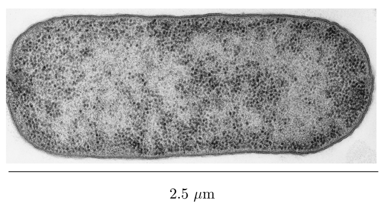
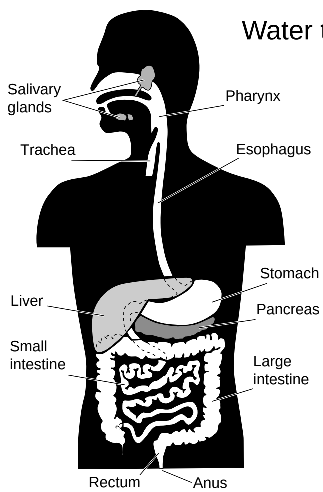



Figure 1.1

## Water transport in digestive system




Daily traffic

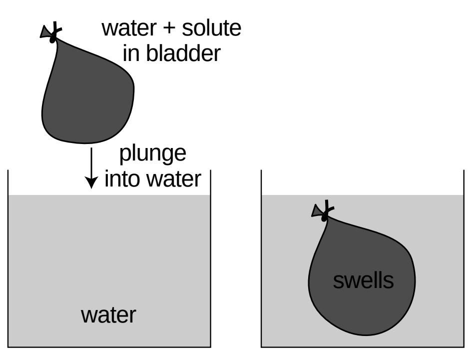
- 800 g food + 1.2 L water ingested daily
- 1.5 L saliva
- 2 L gastric secretions
- 0.5 L bile
- 1.5 L pancreatic secretions
- 1.5 L intestinal secretions

(1.5 L saliva through 1.5 L intestinal secretions: 7 L digestive fluids)

15 pounds of water (10% of body weight) secreted and reabsorbed daily

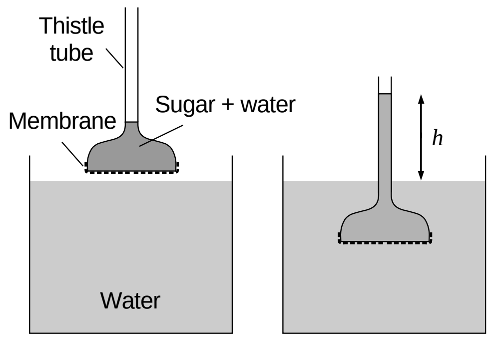
## Osmosis Observations

Henri Dutrochet (early 1800s)
- first described phenomenon and called it osmosis
- developed first osmometer: animal bladder filled with test solution, plunge into water, swells, turgid
- pressure greater for solutions with more solute



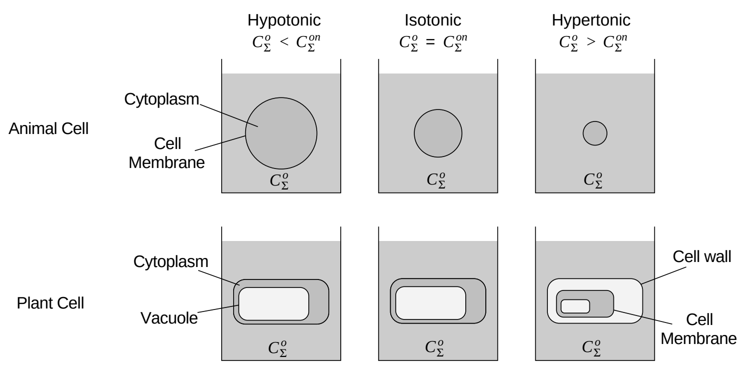
Wilhelm Pfeffer (mid 1800s)

- osmosis can be stopped with hydraulic pressure
- thistle tube + animal bladder (or artificial membrane by late 1800s)
  - water flows in direction to equalize sugar concentration
  - hydraulic pressure develops
  - flow stops when osmotic pressure = hydraulic pressure
- pressure proportional to concentration of solute
- pressure increases slightly with temperature




Hugo de Vries (late 1800s)

- studied osmosis in cells
- animal cell can shrink or swell depending on concentration
- isotonic (same "tension" as in cell's normal environment)
- plasmolysis – plant cell membrane separates from cell wall
- except for salts, plasmolysis occurs at same MOLAR concentration
  (does not depend on chemical properties of solute)
  → colligative property (freezing point depression, boiling point elevation)
- salts are different: ratios of small integers




Henricus van't Hoff (1886)

- formulated mathematical law
- count number of particles in volume V
- measure temperature T
- osmotic pressure = pressure produced by gas with same
  number of particles, same volume, and same pressure

van't Hoff's Law

$$\pi(x,t) = R\,T \sum_n C_n(x,t)$$

osmotic pressure: $\pi(x,t)$

molar gas constant 8.314 J/(mol·K): $R$

absolute temperature: $T$

total solute concentration: $\sum_n C_n(x,t)$

- salts are different

Svante Arrhenius (1884)

- PhD (age 25): dissolution of salts into ions
- $NaCl \rightarrow Na^{+} + Cl^{-}$ ($\therefore$ conducts electricity)
- count ions as separate particles
  $\rightarrow$ van' t Hoff' s law works for salts as well

$$\underbrace{\pi(x,t)}_{\text{osmotic pressure}} = R\,T \sum_n C_n(x,t) = R\,T\,\underbrace{C_\Sigma(x,t)}_{\text{osmolarity}}$$

osmotic pressure [ Pa = N/m$^2$ ]

osmolarity [ osmol/m$^3$ ]

Summary

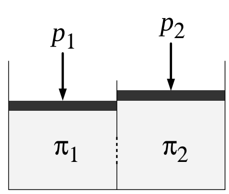
- Osmosis can be characterized by a pressure that is given by van' t Hoff' s Law:

$$\pi = R\,T \sum_n C_n = R\,T\,C_\Sigma$$

- Osmotic pressures cause water to flow
  ... similar to the flow caused by hydraulic pressure
  ... but in the opposite direction!

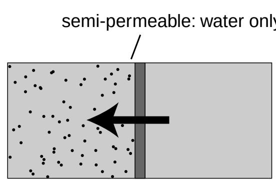
- Water flow stops when osmotic pressure difference is equal to the hydraulic pressure difference:

$$p_1 - p_2 = \pi_1 - \pi_2$$

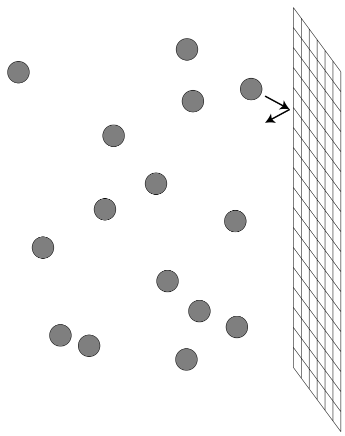



Controversy

- no question that van' t Hoff' s law is true
- but why?




- why should water go TOWARD the solute?
- large osmotic pressure ATTRACTS water!



solute collides with mesh

mesh exerts force on solute
→ changes solute momentum

solute collides with solvent
→ transfers momentum to solvent

change in solvent momentum
is equivalent to a hydraulic pressure

change in hydraulic pressure
= change in osmotic pressure

## Macroscopic laws of solvent transport: hydraulic case

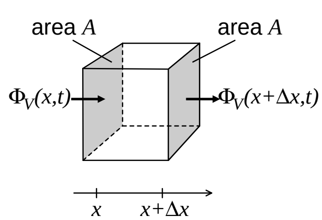
### Darcy' s Law: flow through porous medium

$$\underbrace{\Phi_V(x,t)}_{\text{solvent flux}} = -\overset{\text{hydraulic permeability}}{\kappa}\ \underbrace{\frac{\partial p(x,t)}{\partial x}}_{\text{hydraulic pressure gradient}}$$

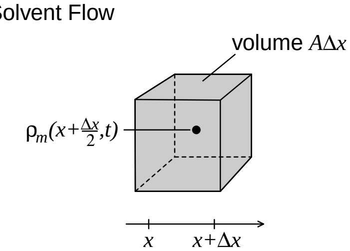
- analogous to Fick' s law for diffusion, Ohm' s law for electrical current, Fourier' s law for heat flow
- units for solvent flux different from units for solute flux

$$\phi_n\ \left[\frac{\text{mol}}{\text{m}^2\text{s}}\right] \qquad \Phi_V\ \left[\frac{\text{m}^3}{\text{m}^2\text{s}} = \frac{\text{m}}{\text{s}}\right] \qquad p\ \left[\text{Pa} = \frac{\text{N}}{\text{m}^2}\right]$$

## Continuity Equation for Solvent Flow




Net mass of solvent entering through edges during $(t,t+\Delta t)$




Change in mass of solvent in volume from $t$ to $t+\Delta t$

$$\underbrace{\rho_m(x,t+\tfrac{\Delta t}{2})\,\Phi_V(x,t+\tfrac{\Delta t}{2})\,A\,\Delta t}_{\text{left}} - \underbrace{\rho_m(x+\Delta x,t+\tfrac{\Delta t}{2})\Phi_V(x+\Delta x,t+\tfrac{\Delta t}{2})\,A\,\Delta t}_{\text{right}} = \underbrace{\rho_m(x+\tfrac{\Delta x}{2},t+\Delta t)\,A\,\Delta x}_{\text{time } t+\Delta t} - \underbrace{\rho_m(x+\tfrac{\Delta x}{2},t)\,A\,\Delta x}_{\text{time } t}$$

↑ equal if solvent is neither created nor destroyed

$$\frac{\rho_m(x,t+\frac{\Delta t}{2})\Phi_V(x,t+\frac{\Delta t}{2}) - \rho_m(x+\Delta x,t+\frac{\Delta t}{2})\Phi_V(x+\Delta x,t+\frac{\Delta t}{2})}{\Delta x} = \frac{\rho_m(x+\frac{\Delta x}{2},t+\Delta t) - \rho_m(x+\frac{\Delta x}{2},t)}{\Delta t}$$

Take limit as $\Delta x \to 0$ and $\Delta t \to 0$

$$-\frac{\partial}{\partial x}\Big[\rho_m(x,t)\Phi_V(x,t)\Big] = \frac{\partial \rho_m(x,t)}{\partial t}$$

Macroscopic laws of water transport (hydraulic case):

- Darcy's law:

$$\Phi_V(x,t) = -\kappa \frac{\partial p(x,t)}{\partial x}$$

- Continuity

$$-\frac{\partial}{\partial x}[\rho_m(x,t)\Phi_V(x,t)] = \frac{\partial \rho_m(x,t)}{\partial t}$$

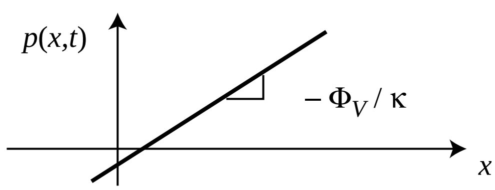
Assume water is incompressible

$\rightarrow$ $\rho_m(x,t) = \rho_o =$ constant in space and time

$$\rightarrow \quad -\rho_o \frac{\partial}{\partial x}[\Phi_V(x,t)] = 0$$

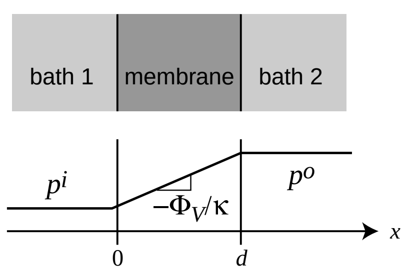
$\rightarrow$ $\Phi_V(x,t) =$ constant in space

$\rightarrow$ $p(x,t) =$ linear function of space




very much like SS case for diffusion
but always true here since water is incompressible

Water flow through thin membrane
(hydraulic)




$$\frac{p^o(t) - p^i(t)}{d} = -\frac{\Phi_V(t)}{\kappa}$$

$$\Phi_V(t) = \mathcal{L}_V\,(\,p^i(t) - p^o(t)\,)\ ; \quad \mathcal{L}_V = \kappa / d$$

hydraulic conductivity

analogous to Fick's law for membranes

$$\phi_n(t) = P_n\,(c_n^i(t) - c_n^o(t))\ ; \quad P_n = \frac{D_n k_n}{d}$$