Quantitative Physiology: Cells and Tissues **Lecture 2: September 8, 2006**

(Volume 1: 3.1-3.1.3; 3.2-3.2.2)

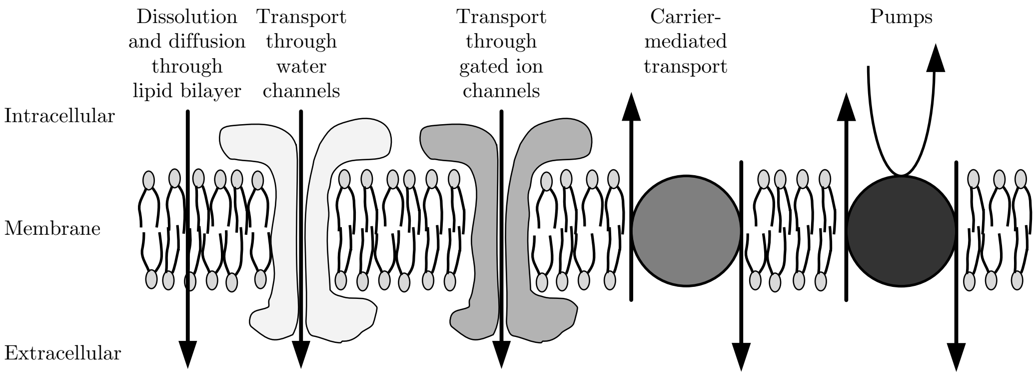
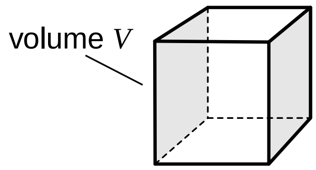


Figure 2.19

Analogous flux laws

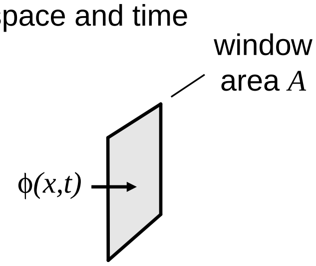
Fourier:	heat flux	α	$\dfrac{d\,(\text{temperature})}{dx}$
Ohm:	current	α	$\dfrac{d\,(\text{potential})}{dx}$
Fick:	substance flux	α	$\dfrac{d\,(\text{concentration})}{dx}$

Concentration at a point
in space and time



$$\text{concentration } c(x,t) = \lim_{V \to 0} \frac{\text{amount of substance in } V}{V}$$

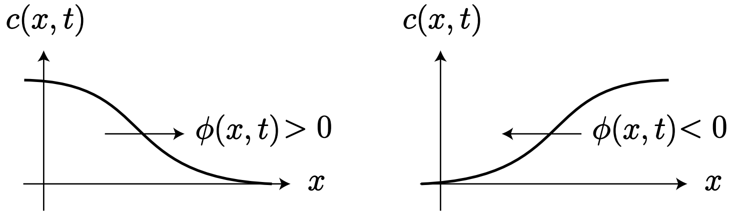
Flux at a point
in space and time



$$\text{flux } \phi(x,t) = \lim_{\substack{A \to 0 \\ \Delta t \to 0}} \frac{\text{amount of substance flowing through test window } A \text{ in } \Delta t}{A \Delta t}$$

Fick's First Law

$$\phi(x,t) = -D\frac{\partial c(x,t)}{\partial x}$$

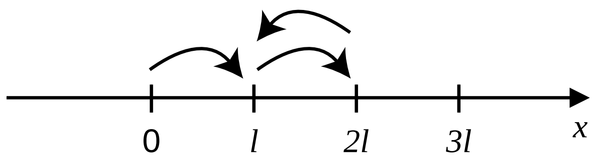


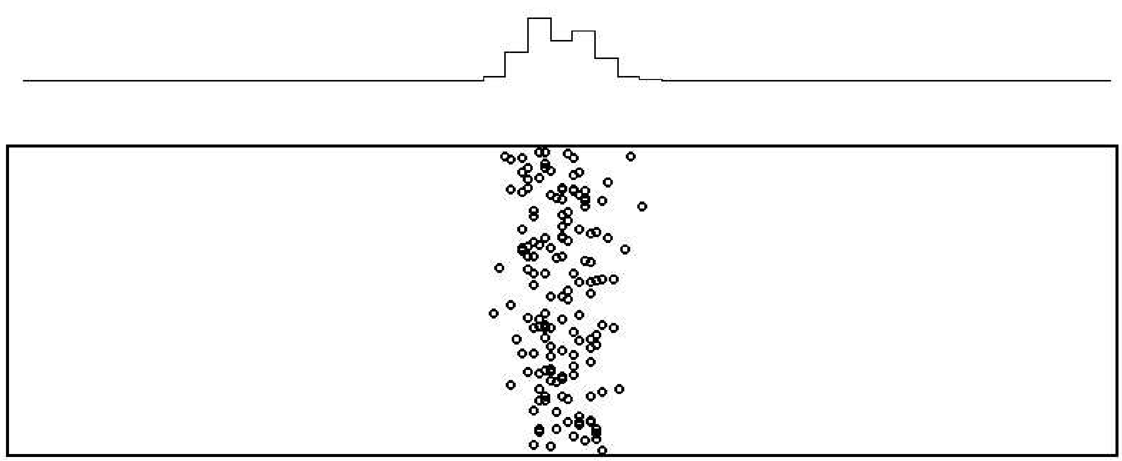
Random Walk Model

- number of solute particles << number of solvent particles
- motion of solute determined by collisions with solvent (ignore solute-solute interactions)
- focus on 1 solute particle, assume motions of others are statistically identical

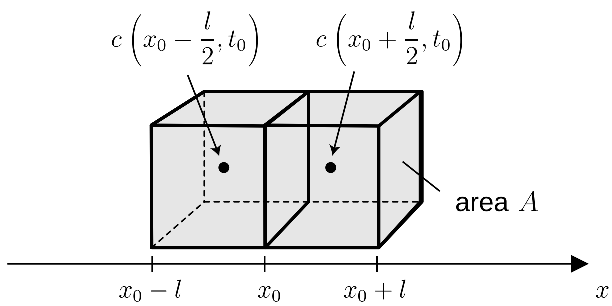
Every τ seconds, solute particle gets hit by solvent particle.

In response, solute particle is equally likely to move $+l$ or $-l$.
τ = mean free time; l = mean free path





Random Walk Model → Fick's First Law



$$\phi^+(x_0, t_0) = \frac{1}{2}\left(\frac{c(x_0 - \frac{l}{2}, t_0)\ Al}{A\tau}\right) \qquad \phi^-(x_0, t_0) = \frac{1}{2}\left(\frac{c(x_0 + \frac{l}{2}, t_0)\ Al}{A\tau}\right)$$

$$\phi(x_0, t_0) = \frac{l}{2\tau}\left(c(x_0 - \frac{l}{2}, t_0) - c(x_0 + \frac{l}{2}, t_0)\right)$$

$$\phi(x_0, t_0) = -\frac{l^2}{2\tau}\left(\frac{c(x_0 + \frac{l}{2}, t_0) - c(x_0 - \frac{l}{2}, t_0)}{l}\right)$$

$$\phi(x, t) \approx -\left(\frac{l^2}{2\tau}\right)\frac{\partial c(x, t)}{\partial x}$$