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Quantitative Physiology: Cells and Tissues

Homework Assignment #6

Issued: October 26, 2006

Due: November 2, 2006

Part of this week's homework assignment is to submit the final draft of your microfluids report.

Exercise 1. The resistance of conductors is described by several different related variables: the resistivity, ρ ; the resistance per unit length, r ; the resistance of a unit area, R ; the total resistance, \mathcal{R} . Describe the meaning of each of these resistances and give their units.

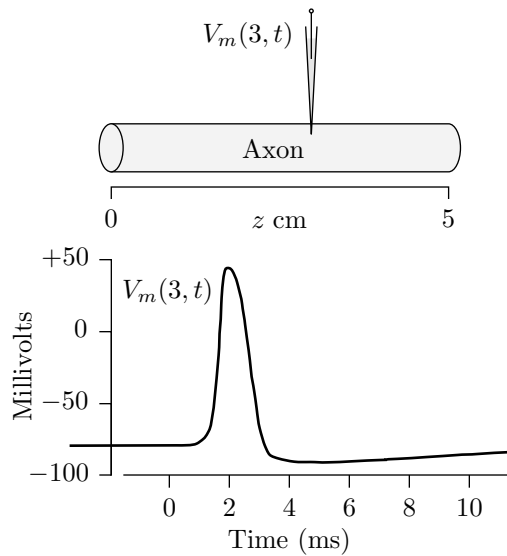
Exercise 2. An electrically large, cylindrical cell has a radius of $100 \mu\text{m}$. The internal longitudinal current is constant in time and its spatial dependence is

$$I_i(z) = -e^{5z}, \text{ for } z < 0$$

where $I_i(z)$ has units of μA and z has units of cm. There are no external currents for $z < 0$.

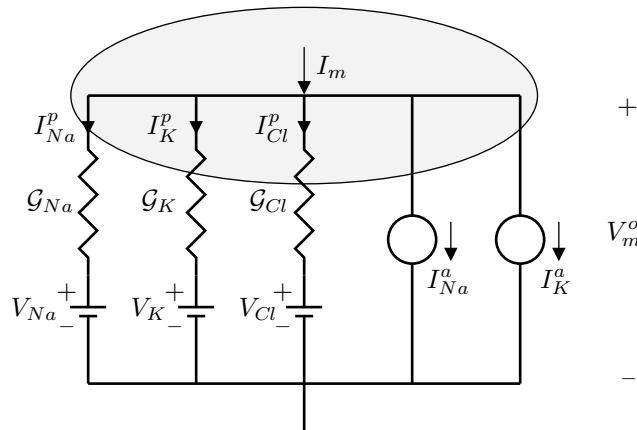
- a) Sketch $I_i(z)$ versus z for $z < 0$. Also make a schematic diagram of the cell and sketch $I_i(z)$ on the cell using arrows whose direction indicates the direction of the current and whose length indicates the magnitude of the current. Sketch the current every millimeter from $-0.6 < z < 0$.
- b) Determine the external longitudinal current $I_o(z)$ for $z < 0$. Sketch $I_o(z)$ versus z for $z < 0$. On the same schematic diagram of the cell used in part a), sketch $I_o(z)$ on the cell using arrows whose direction indicates the direction of the current and whose length indicates the magnitude of the current. Sketch the current every millimeter from $-0.6 < z < 0$.
- c) Determine the current per unit length, $K_m(z)$ for $z < 0$. Sketch $K_m(z)$ versus z for $z < 0$. On the same schematic diagram of the cell used in part a), sketch $K_m(z)$ on the cell using arrows whose direction indicates the direction of the current per unit length and whose length indicates the magnitude of the current per unit length. Sketch the current per unit length every millimeter from $-0.6 < z < 0$.

Exercise 3. An action potential, $V_m(z, t)$, propagates at constant velocity along an unmyelinated axon and is recorded as a function of time at position $z = 3 \text{ cm}$ as shown in the following figure.



- a) Assume the action potential propagates in the $+z$ -direction with conduction velocity $\nu = 2$ cm/ms.
- Sketch $V_m(5, t)$, i.e., sketch the dependence of V_m on t at position $z = 5$ cm.
 - Sketch $V_m(z, 5)$, i.e., sketch the dependence of V_m on z at time $t = 5$ ms.
- b) Repeat part a) for an action potential propagating in the $-z$ -direction with conduction velocity $\nu = 2$ cm/ms.

Problem 1. Consider the model of a cell shown in the following figure.



The cell has channels for the passive transport of sodium, potassium, and chloride as well as a pump that actively transports sodium out of the cell and potassium into the cell. The pump ratio is $I_{Na}^a/I_K^a = -1.5$. The following table shows the intracellular and extracellular concentrations, Nernst equilibrium potentials, and conductance ratios for sodium and potassium. Some information is also given for chloride; blank entries represent unknown quantities.

	c_n^i	c_n^o	V_n	G_n/G_K
	(mmol/L)		(mV)	
Na ⁺	10	140	+68	0.1
K ⁺	140	10	-68	1
Cl ⁻		150		1

The cell also contains impermeant intracellular ions. Assume that the cell is in equilibrium at $t = 0$, i.e., assume that at $t = 0$ the cell has reached a condition for which all solute concentrations, the cell volume, and the membrane potential are constant.

- a) Choose one of the following statements and explain why it is true.
 - i) The cell resting potential depends on \mathcal{G}_{Cl} .
 - ii) The cell resting potential depends on V_{Cl} .
 - iii) The cell resting potential depends on both \mathcal{G}_{Cl} and V_{Cl} .
 - iv) The cell resting potential does not depend on \mathcal{G}_{Cl} .
 - v) The cell resting potential does not depend on V_{Cl} .
 - vi) The cell resting potential does not depend on either \mathcal{G}_{Cl} or V_{Cl} .
- b) Determine V_m^o .
- c) At $t = 0$, the external concentration of chloride is reduced from 150 mmol/L to 50 mmol/L by substituting an isosmotic quantity of an impermeant anion for chloride. Assume that the concentrations of sodium and potassium both inside and outside the cell remain the same and that the volume of the cell does not change.
 - i) Determine $V_m^o(0^+)$, the value of the membrane potential immediately after the change in solution. You may ignore the effect of the membrane capacitance.
 - ii) Determine $V_m^o(\infty)$, the value of the membrane potential after the cell has equilibrated.
 - iii) Determine $c_{Cl}^i(\infty)$, the intracellular chloride concentration after the cell has equilibrated.
 - iv) Give a physical interpretation of your results in i), ii), and iii).
 - v) Discuss the validity of the assumptions that the sodium and potassium concentrations in the cell are constant and that the volume does not change.

Problem 2. A squid axon of diameter $500 \mu\text{m}$ is placed in an insulated trough filled with sea water (a cross-section through the trough is shown in Figure 1). Assume that the resistivity of sea water

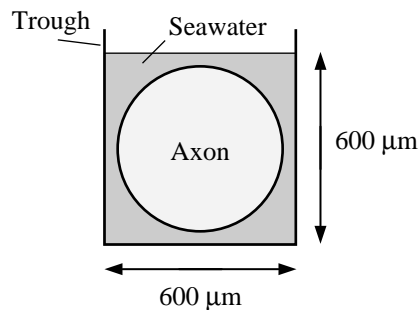
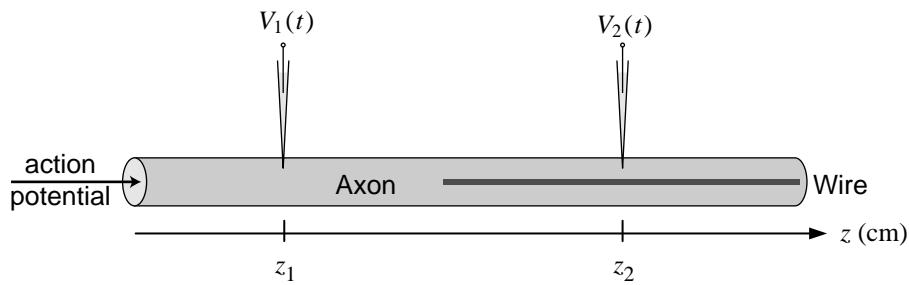


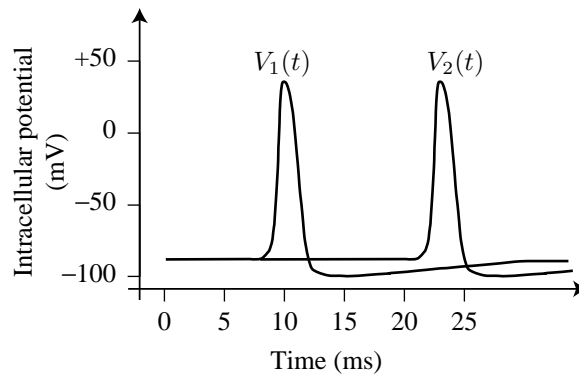
Figure 1: Cross-section of an axon in a trough of sea water.

and axoplasm are both $25 \Omega\text{-cm}$. The peak-to-peak amplitude of the action potential measured across the membrane is 100 mV. What is the peak-to-peak amplitude of the action potential measured with an extracellular electrode (the reference electrode is placed on a remote, inactive end of the axon). In this calculation, assume that the core conductor model is valid.

Problem 3. A fine platinum wire with a resistance per unit length of $130 \Omega/\text{cm}$ is inserted inside a portion of a squid axon as illustrated below.



The wire is so thin that its volume can be ignored. The axon ($500 \mu\text{m}$ diameter) is electrically stimulated to produce a propagated action potential traveling in the $+z$ direction. The action potential is recorded at two intracellular sites: $V_1(t)$ is recorded at $z = z_1$ and $V_2(t)$ is recorded at $z = z_2$. The distance between the stimulus electrode (not shown) and z_1 is 2 cm. Results are shown in the following figure.



The resistivity of the axoplasm of this axon is $23 \Omega\text{-cm}$. The resistance per unit length of the external solution is $1.2 \Omega/\text{cm}$. The wire begins at some location between z_1 and z_2 , but the exact position of the beginning is not known and should not be used in any of your calculations.

- a) Determine the instantaneous speed of the action potential as its peak passes the point $z = z_1$.
- b) Determine the instantaneous speed of the action potential as its peak passes the point $z = z_2$.
- c) Sketch the extracellular potential as a function of space (z) that results at the time that the peak of the action potential passes the point $z = z_1$. Include distances $z_1 - 4 < z < z_1 + 4$ cm. Indicate the scale for the y axis. Describe the important features of this plot.
- d) Sketch the extracellular potential as a function of space (z) that results at the time that the peak of the action potential passes the point $z = z_2$. Include distances $z_2 - 4 < z < z_2 + 4$ cm. Indicate the scale for the y axis. Describe the important features of this plot.