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Quantitative Physiology: Cells and Tissues

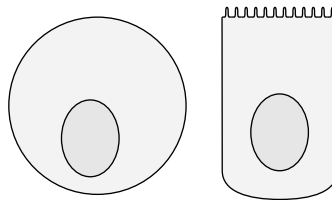
Homework Assignment #3

Issued: September 21, 2006

Due: September 28, 2006

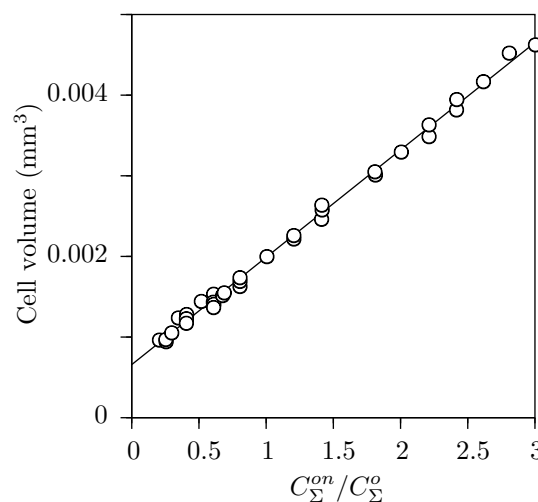
Exercise 1. What is a *colligative property* of a solution? List 3 colligative properties of solutions.

Exercise 2. The following figure shows schematic diagrams of two cells that have the same volume but quite different shapes.



One cell (left panel) is spherical, the other is approximately cylindrical but contains a large number of microvilli. The cell membranes have the same hydraulic conductivity to water. If the two cells are subjected to the same decrease in extracellular osmolarity, which cell swells more rapidly? Explain.

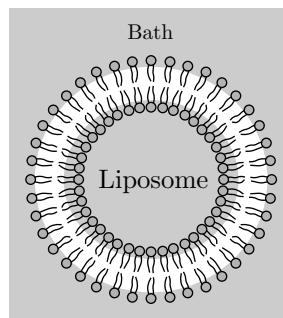
Exercise 3. A cylindrical cell is bathed in solutions of a single impermeant substance that does not ionize. For each solution, the concentration C_{Σ}^o of the impermeant substance and the equilibrium cell volume are measured and the results are shown in the following figure where C_{Σ}^{on} represents the isotonic concentration.



a) Estimate the isotonic volume of the cylindrical cell.

- b) Assume that the cell volume \mathcal{V}_c contains a portion \mathcal{V}^i that consists of water that is osmotically active and a portion \mathcal{V}'_c that is osmotically inactive. Estimate \mathcal{V}'_c and explain the basis of your estimate.
- c) Assume that $C_{\Sigma}^{on} = 200 \text{ mosm/L}$ and that the cell membrane is impermeable to NaCl. Estimate the equilibrium cell volume that would result if the cell were bathed in a solution of NaCl with concentration 200 mmol/L. Explain your reasoning.
- d) Assume that the cell bath is changed from an isotonic solution to a 200 mmol/L NaCl solution at time $t = 0$. Sketch the time course for equilibration of the cell volume indicating numerical values where possible. List the physical parameters of the cell that influence the time course.

Exercise 4. A large fraction of the molecules in a cell membrane are phospholipids, which have a hydrophilic head and hydrophobic tails. When purified phospholipids are added to a saline solution, the phospholipids self assemble into a variety of stable structures, one of which is called a liposome.

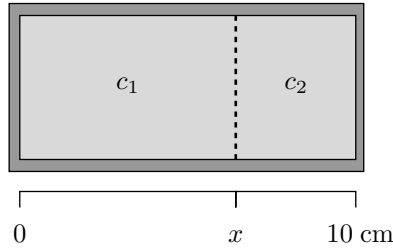


Liposomes have saline interiors and exteriors that are separated by a phospholipid bilayer, much like a biological cell, as illustrated above. Liposomes can be used as artificial cells to test theories about membrane transport. Assume that a liposome is created in a solution that contains 100 mmol/L of a solute A that cannot permeate the phospholipid bilayer. Assume that the volume of the liposome is 1 pL and that the internal solution is the same as the bath (hereafter called bath 1). Assume that both water and a second solute B can permeate the phospholipid bilayer.

Part a. We wish to transfer the liposome from bath 1 to a second bath where the liposome will initially shrink but then come to a steady-state with a volume of 2 pL. Is it possible to combine solutes A and B to compose such a second bath? If yes, what concentrations of A and B should be in the second bath? If no, explain why not.

Part b. We wish to transfer the liposome from bath 1 to a third bath where the liposome will initially swell but then come to a steady-state with a volume of 0.5 pL. Is it possible to combine solutes A and B to compose such a third bath? If yes, what concentrations of A and B should be in the third bath? If no, explain why not.

Problem 1. A volume element with constant cross-sectional area A has rigid walls and is divided into two parts by a rigid, semipermeable membrane that is mounted on frictionless bearings so that the membrane is free to move in the x -direction as shown in the following figure.



The semipermeable membrane is permeable to water but not to the solutes (glucose or NaCl). At $t = 0$, solute 1 is added to side 1 to give an initial concentration of $c_1(0)$ and solute 2 is added to side 2 to give an initial concentration of $c_2(0)$. Concentrations are specified as the number of millimoles of glucose or NaCl per liter of solution. The initial position of the membrane is $x(0)$. For each of the initial membrane positions and concentrations given in the following table, find the final (equilibrium) values of the membrane position $x(\infty)$, and the concentrations, $c_1(\infty)$ and $c_2(\infty)$. State your assumptions.

Solutes		Initial Values			Final Values		
1	2	$c_1(0)$ mmol/L	$c_2(0)$ mmol/L	$x(0)$ cm	$c_1(\infty)$ mmol/L	$c_2(\infty)$ mmol/L	$x(\infty)$ cm
Glucose	Glucose	0	10	5			
Glucose	Glucose	10	10	8			
Glucose	Glucose	30	10	5			
Glucose	NaCl	20	10	2			
Glucose	NaCl	80	10	2			
NaCl	NaCl	30	10	5			

Problem 2. A spherical cell is subjected to four different aqueous solutions of impermeant solutes and its equilibrium radius is measured as shown in Figure 1. The isotonic radius is $80 \mu\text{m}$. The

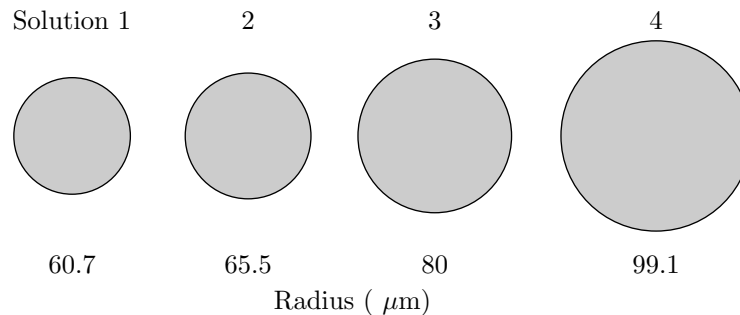
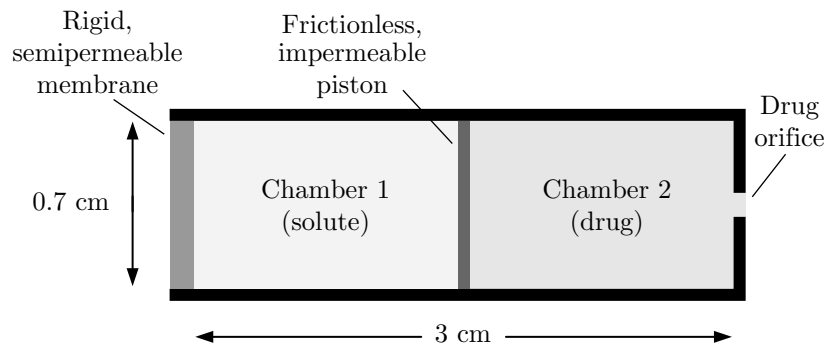


Figure 1: A spherical cell placed in four different solutions of impermeant solutes. The circles represent cross-sectional diagrams of the cell, and the numbers below the circles indicate the measured radii.

four solutions are: 150 mmol/L NaCl, 200 mmol/L CaCl_2 , 800 mmol/L sucrose, and 150 mmol/L xylose. The latter two solutes are sugars. You may assume that the intracellular quantity of solute does not change during these measurements.

- a) Determine the compositions of Solutions 1-4.
- b) Find the total quantity of intracellular solute.
- c) What fraction of the isotonic volume of the cell is due to osmotically active water?

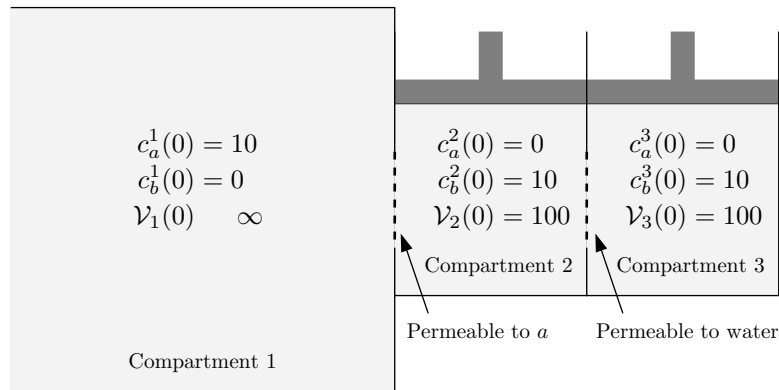
Problem 3. The following figure shows the design of a miniature pump that can be implanted in the body to deliver a drug. No batteries are required to run this pump!



The pump contains two cylindrical chambers filled with incompressible fluids: the two chambers together have a length of 3 cm and a diameter of 0.7 cm. Chamber 1 is filled with a solution whose concentration is 10 mol/L; the osmolarity of this solution greatly exceeds that of body fluids. Chamber 2 is filled with the drug solution. The two chambers are separated by a frictionless, massless, and impermeable piston. The piston moves freely and supports no difference in hydraulic pressure between the chambers; the piston allows no transport of water, solute or drug between chambers. The pump walls are rigid, impermeable and cylindrical with an orifice at one end for delivering the drug and a rigid, semipermeable membrane at the other end. The orifice diameter is sufficiently large that the hydraulic pressure drop across this orifice is negligible and sufficiently small so that the diffusion of drug through the orifice is also negligible. The semipermeable membrane is permeable to water only, and not permeable to the solute. Assume that $T = 300$ K.

- a) Provide a discussion of 50 words or fewer for each of the following:
 - i) What is the physical mechanism of drug delivery implied by the pump design?
 - ii) What is(are) the source(s) of energy for pumping the drug?
 - iii) Assume there is an adequate supply of drug in the pump for the lifetime of the implanted subject and that it is necessary to provide a constant rate of drug delivery. Which fundamental factors limit the useful lifetime of this pump in the body?
- b) When implanted in the body, the pump delivers the drug at a rate of $1 \mu\text{L/h}$. Find the value of the hydraulic conductivity, \mathcal{L}_V , of the semipermeable membrane.

Problem 4. Three compartments filled with aqueous solutions are separated by semipermeable membranes as shown in the following figure.



Two non-electrolyte solutes *a* and *b* are contained in the solutions. Initial concentrations of each solute (in mmol/L) and initial volumes (in cm³) are given in the figure. The membrane separating compartments 1 and 2 is permeable to solute *a* only and not permeable either to solute *b* or to water. The membrane separating compartments 2 and 3 is permeable to water only and not permeable to either solutes *a* or *b*. The volume of Compartment 1, \mathcal{V}_1 , is much larger than that of the other compartments. You may assume that it has an infinite volume. Compartments 2 and 3 contain pistons with frictionless bearing surfaces. Therefore, the hydraulic pressure in these compartments is zero.

- a) Is the system in diffusive equilibrium at $t = 0$? Explain.
- b) Is the system in osmotic equilibrium at $t = 0$? Explain.
- c) Determine the equilibrium concentrations ($c_a^1(\infty)$, $c_b^1(\infty)$, $c_a^2(\infty)$, $c_b^2(\infty)$, $c_a^3(\infty)$, and $c_b^3(\infty)$) and volumes ($\mathcal{V}_2(\infty)$ and $\mathcal{V}_3(\infty)$).

Problem 5. Each of 6 identical cells are put into separate baths that contain an isotonic solution of impermeant uncharged solute with concentration C . At time $t = 0$, a different test solute is added to each bath. None of the solutes are salts so none of the solutes dissociate. At time $t = 15$ seconds, each bath is changed back to the original isotonic solution. For the different test solutes A-F, the cell's volume response shows characteristic differences as shown in Figure 2. Assume that the cell membranes transport water and that the transport of water and solute are uncoupled, i.e., the flow of water through the membrane does not affect diffusion of solute and the diffusion of solute does not affect the flow of water.

- a) Which, if any, of the volume responses A-F correspond to results that would be obtained with an impermeant test solute? Explain!
- b) Assume that the concentration C_x is the same for A, B, and C. Which of these test solutes has the highest membrane permeability? Explain!
- c) Assume that the concentration C_x is the same for A, B, and C. Find the numerical value of C_x/C . Explain!
- d) Assume that C_x is the same for B and F. Is the permeability of test solute B greater than or less than that for test solute F? Explain!

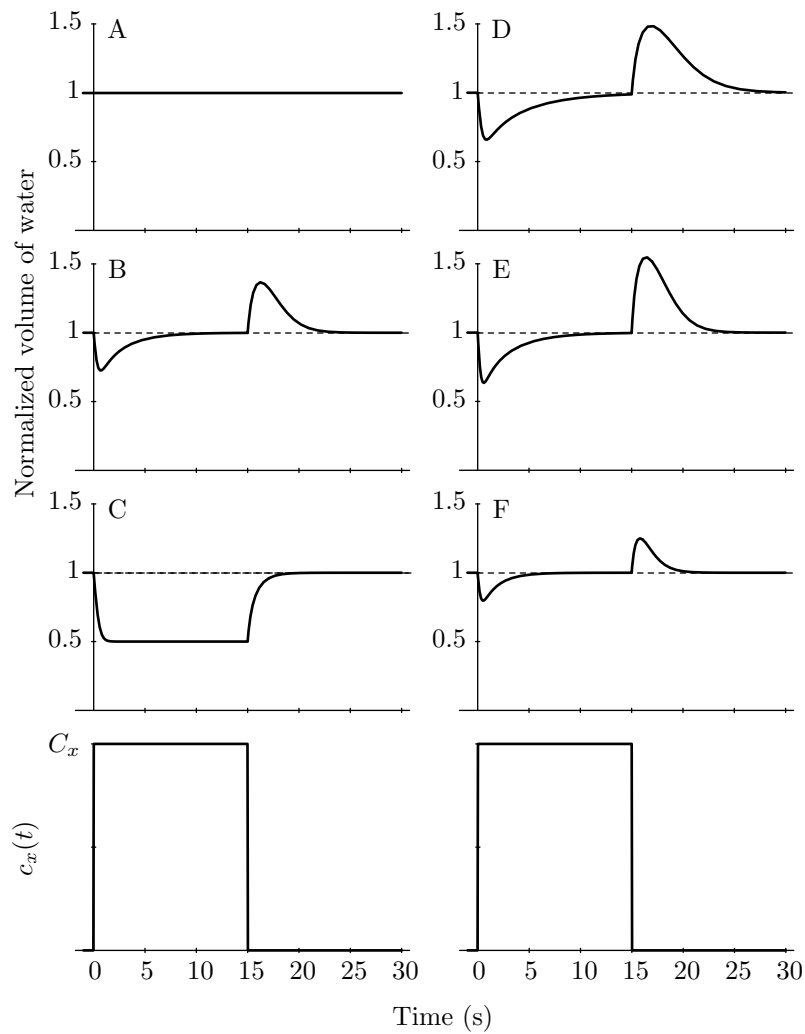


Figure 2: Response of a cell to 6 nonelectrolyte solutes. The normalized volume represents the volume of cell water normalized to its isotonic value.

- e) Assume that the same species of test solutes is used to obtain B and E. For which of these responses is C_x larger? Explain!
- e) The *time course* of the onset of the response at $t = 0$ appears to be about the same in B-F. Why?