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 2.791J/2.794J/6.021J/6.521J/20.370J/20.470J/HST.541J  
**Quantitative Physiology: Cells and Tissues**

**Homework Assignment #2**

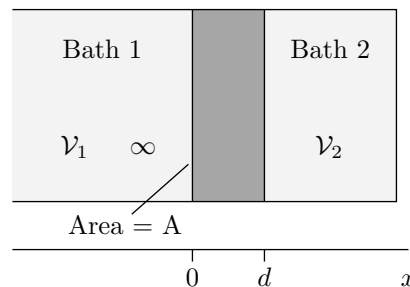
Issued: September 14, 2006  
 Due: September 21, 2006

**Announcements**

Part of this week's homework assignment is to write a proposal for your microfluidics project. See the 6.021J website for more information.

**Exercise 1.** Two time constants are involved in two-compartment diffusion through a membrane: the steady-state time constant of the membrane ( $\tau_{ss}$ ) and the equilibrium time constant for the two compartments ( $\tau_{eq}$ ). Without the use of equations, describe these two time constants.

**Exercise 2.** Two solutions are separated by a membrane of thickness  $d$  and surface area  $A$ , as shown in the following figure.



The membrane:solution partition coefficient  $k_n = 2$ . Bath #1 has an infinite volume and has a concentration of solute  $n$  of  $C$ . Bath #2 has volume  $\mathcal{V}_2$  and a concentration of solute  $n$  that is  $C/4$ .

**Part a.** Assume that the membrane has reached steady state at  $t = 0$ .

**Part a1.** Let  $c_n(x, t)$  represent the concentration of solute  $n$  at position  $x$  and time  $t$ . Sketch  $c_n(x, 0)$  for values of  $x$  that include the membrane and the two baths.

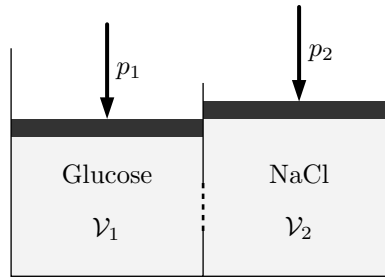
**Part a2.** Is the system in equilibrium at  $t = 0$ ? Explain.

**Part b.** Let  $P_n$  represent the steady state permeability of the membrane to solute  $n$ .

**Part b1.** Assume that the membrane can be treated as a *thin membrane* and that the baths are “well stirred.” The concentration of solute  $n$  in Bath #2  $c_n^2(t) = C/4$  for  $t < 0$ . Sketch  $c_n^2(t)$  for  $t > 0$ .

**Part b2.** Assume that the membrane is too thick to be treated as a thin membrane. Describe how the plot of  $c_n^2(t)$  versus  $t$  would differ from the plot in part b1.

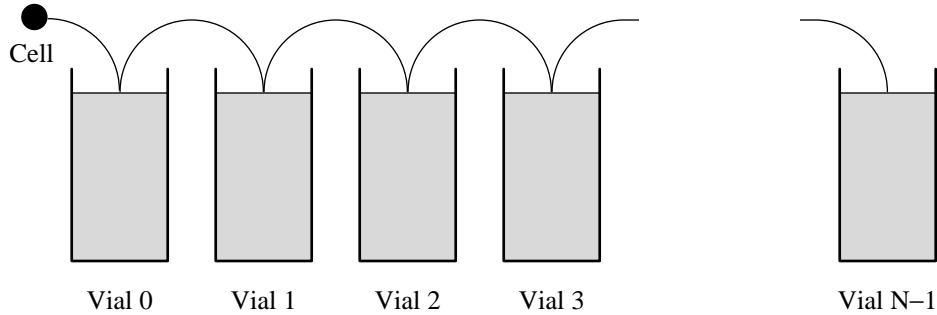
**Exercise 3.** The following figure shows an experimental apparatus for testing a semipermeable membrane.



Volumes  $\mathcal{V}_1$  and  $\mathcal{V}_2$  contain well-stirred aqueous solutions of glucose and NaCl, respectively. These volumes are separated by a membrane that is permeable only to water. Assume that the pistons are ideal — i.e. that they are frictionless, and faithfully transmit the pressures  $P_1$  and  $P_2$  to  $\mathcal{V}_1$  and  $\mathcal{V}_2$ , respectively. Also assume that effects of gravity are negligible.

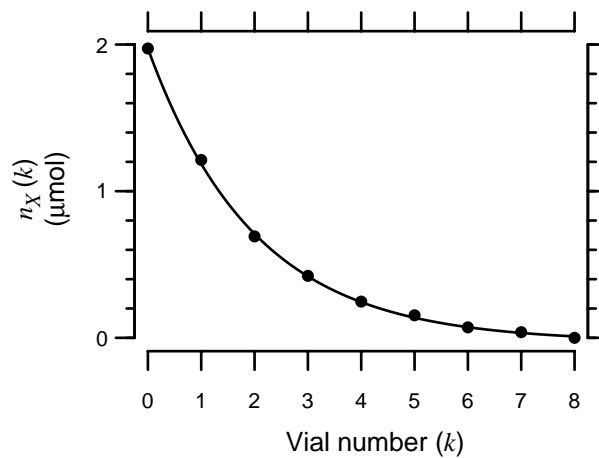
- (a) When the system is in equilibrium, what will be the relation between hydraulic pressures ( $P_1$  and  $P_2$ ) and solute concentrations in each compartment?
- (b) Volumes  $\mathcal{V}_1$  and  $\mathcal{V}_2$  are initially equal, with  $\mathcal{V}_1 = \mathcal{V}_2 = 1$  L. At time  $t = 0$ , the concentration of glucose in compartment 1 is 0.01 mol/L and the concentration of NaCl in compartment 2 is 0.01 mol/L. If  $P_1 = P_2$ , what is the final volume of compartment 2?

**Problem 1.** An experiment is performed to determine the permeability,  $P_X$ , of the membrane of a cell to solute  $X$ . The cell is spherical and has a radius of  $72 \mu\text{m}$ . It is placed in a solution containing solute  $X$  for a sufficient time to load the cell with  $N_X$  moles of  $X$ . A set of identical vials containing identical solutions that do not contain the solute  $X$  are prepared. The cell is then immersed successively in the series of these vials for  $T = 10$  minutes per vial, i.e., 10 minutes in vial 0, followed by 10 minutes in vial 1, followed by 10 minutes in vial 2, etc., as shown in the following figure.

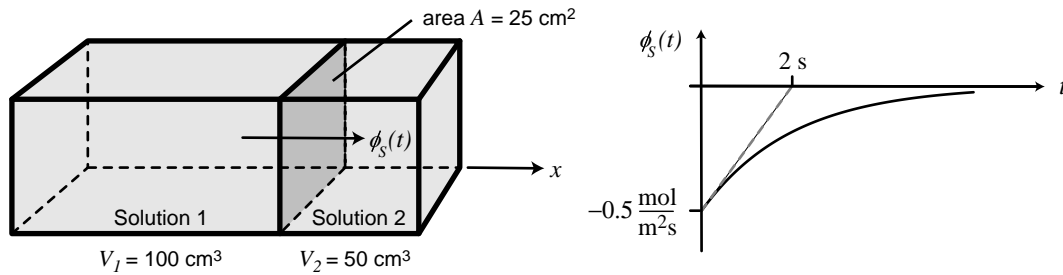


The number of moles of solute  $X$  in vial  $k$ , is  $n_X(k)$ . Assume that the volume of the cell is constant and negligible compared with the volume of a vial, and hence, the concentration of solute in a vial is always negligible compared to that in the cell. The solute permeates the membrane according to Fick's Law for membranes.

- Determine an expression for the number of moles of  $X$  in the cell as a function of time,  $n_X^i(t)$ . You may write this expression in terms of literals such as  $N_X$  and a suitably defined time constant. Assume that the transfer of the cell from vial to vial takes no time.
- Determine an expression for the total quantity of  $X$  in the  $k^{\text{th}}$  vial,  $n_X(k)$  in terms of literals such as  $N_X$ ,  $T$ , and a suitably defined time constant.
- For the measurements shown in figure shown below, determine the numerical values of  $P_X$  and  $N_X$ .



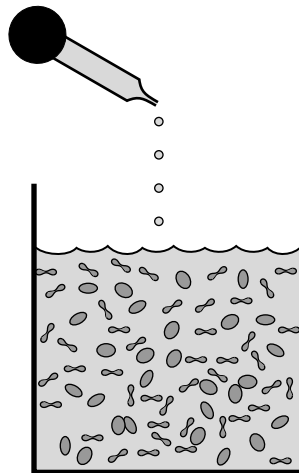
**Problem 2.** Two solutions of an uncharged solute  $S$  have volumes  $V_1 = 100 \text{ cm}^3$  and  $V_2 = 50 \text{ cm}^3$  and are separated by a thin membrane (area  $A = 25 \text{ cm}^2$ ) permeant to  $S$  and impermeant to water.



The flux of  $S$  through the membrane obeys Fick's law for membranes. At time  $t = 0$ , the concentration of  $S$  in solution 1 is  $c_1(0) = 100 \text{ mol/m}^3$ . The initial concentration of  $S$  in solution 2 is not known. The flux of  $S$  through the membrane in the positive  $x$  direction is found to be an exponential function of time as shown in the plot.

- Determine the concentration  $c_1(t)$  of  $S$  in solution 1 and the concentration  $c_2(t)$  of  $S$  in solution 2 as functions of time, assuming that the solutions are well-stirred. Sketch  $c_1(t)$  and  $c_2(t)$  on suitably labeled axes.
- Determine numerical values for the final concentrations of  $S$ :  $c_1(\infty)$  and  $c_2(\infty)$ . If it is not possible to determine numerical values, list the other information that would be needed to determine a numerical value.

**Problem 3.** Glucose is dripped at a constant rate  $R = 2 \mu\text{mol/s}$  into a bath that contains  $10^{12}$  identical red blood cells, as shown in the following figure.



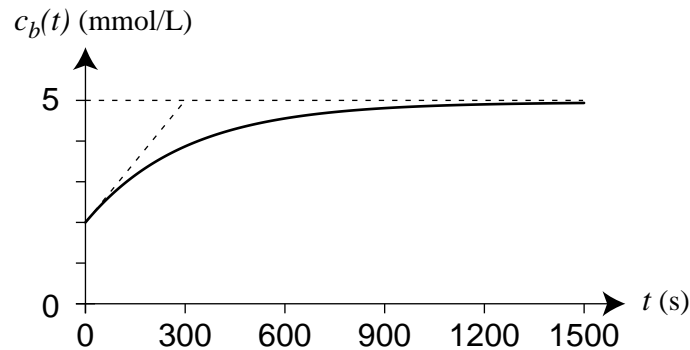
Assume that each red blood cell has a volume  $\mathcal{V}_C = 25 (\mu\text{m})^3$  and a surface area  $\mathcal{A}_C = 80 (\mu\text{m})^2$ , and that neither of these changes over the time interval considered in this problem. Assume that the volume of the bath is 1 L, and that the bath is well stirred. (You may assume that the amount of water dripped into the bath is negligibly small.)

**Part a.** Is the following statement True or False?

If the flux of glucose through each of the cell membranes were constant with time, then the concentration of glucose in the bath,  $c_b(t)$ , would be a linear function of time.

If the truthfulness of this statement cannot be determined from the information provided, describe what additional information is needed.

**Part b.** The concentration of glucose in the bath,  $c_b(t)$ , is found to increase as a function of time  $t$ , as shown in the following plot.



**Part b1.** Determine the flux of glucose through the membrane of each cell at time  $t = 1500$  s. Use our normal convention that outward flux (i.e., flux leaving the cell) is positive and inward flux is negative. Determine the numerical value (or numerical expression) **and units**. If you cannot determine the numerical value from the information provided, describe what additional information is needed.

**Part b2.** Determine the flux of glucose through the membrane of each cell at time  $t = 0$  s. Use our normal convention that outward flux (i.e., flux leaving the cell) is positive and inward flux is negative. Determine the numerical value (or numerical expression) **and units**. If you cannot determine the numerical value from the information provided, describe what additional information is needed.

**Problem 4.** The water content of plants is very high (up to 90% by weight) but this water is in flux; water is absorbed through the roots, rises as sap, and evaporates from the leaves. The total water content of a plant can be replaced many times per hour. The mechanisms that determine water flow include: gravity, osmosis, and capillarity. In this problem we will consider the effect of the gravity and osmosis. Assume that water flow is steady and is due to gravitational, osmotic, and *other* forces (such a capillarity forces). Let the pressure due to gravity be  $p_g$  and the osmotic pressure be  $\pi$ . Let the pressure due to other sources be  $p_o$  in the trees and zero in the medium that is in contact with the roots. Consider two trees:



The General Sherman giant sequoia (*Sequoia gigantea*) which is located in Sequoia National Park in California stands 272 feet high and has a diameter of 36.5 feet at its base; you can drive your car through a tunnel that has been cut through the base. The tree was estimated to be 3,800 years old.

Image from the National Park Service.



The red mangrove (*Rhizophora mangle*) grows in the tropics to a height of as much as 80 feet. It is found in tidal creeks and estuaries and it can grow with its roots in seawater. Yet the sap has the composition of fresh water.

Image from the National Oceanic and Atmospheric Administration.

The compositions of the relevant media are given in below.

Ion	Concentration (mmol/L)			
	Giant sequoia		Red mangrove	
	soil	sap	seawater	sap
$K^+$	1	20	10	20
$Na^+$	2	0	450	0
$Cl^-$	30	2	530	2
$NO_3^-$	3	20	0	20
$Ca^{++}$	7	1	10	1
$Mg^{++}$	8	0	50	0

- Find a numerical bound on  $p_o$  (expressed in atmospheres) such that the sap will rise in each tree.
- Which tree requires the larger value of  $p_o$ ? Explain.